



Carli, Inc.

18 Tanglewood Rd.
Amherst, MA 01002

TEL: (413) 256-4647
FAX: (413) 256-4823

home@fenestration.com
<http://www.fenestration.com>

Technical Report:

TARCOG: MATHEMATICAL MODELS FOR CALCULATION OF THERMAL PERFORMANCE OF GLAZING SYSTEMS WITH OR WITHOUT SHADING DEVICES

October 17, 2006

TABLE OF CONTENTS

1. EXECUTIVE SUMMARY	4
2. INTRODUCTION	4
2.1. Layer	4
2.2. Gap	7
2.3. Glazing System.....	7
3. ISO 15099 ALGORITHMS	10
3.1. GLAZING SYSTEMS CONSISTING OF MONOLITHIC LAYERS	10
3.1.1. Definitions	10
3.1.2. Heat Transfer Calculations	12
3.1.2.1. ENERGY BALANCE	13
3.1.2.2. INITIAL TEMPERATURE DISTRIBUTION	14
3.1.2.3. BOUNDARY CONDITIONS	15
3.1.2.3.1. Outdoor Heat Transfer Coefficients	15
3.1.2.3.2. Indoor Heat Transfer Coefficients	16
3.1.2.3.3. Glazing Cavity Heat Transfer	18
3.1.2.3.4. Thermo-physical Properties of Gases.....	20
3.1.2.3.5. Interaction with the environment	22
3.1.2.4. SOLUTION OF THE SYSTEM OF NON-LINEAR EQUATIONS	24
3.1.2.5. CONVERGENCE CHECKING	26
3.1.3. Calculation of U – factor.....	26
3.1.4. Calculation of Solar Heat Gain Coefficient	27
3.2. GLAZING SYSTEMS WITH LAMINATED LAYERS.....	28
3.2.1. Energy Balance Equations for Laminated Layers	28
3.2.2. System of Equations for Glazing Systems Incorporating Laminated Layers.....	32
3.2.3. Determination of Glazing Surface Temperatures	34
3.2.4. Calculation of U – factor and SHGC for Laminated Layers	34
3.3. GLAZING SYSTEMS WITH SHADING DEVICES	35
3.3.1. Modifications of Basic Energy Balance Equations	35
3.3.1.1. CALCULATION OF GLAZING CAVITY TEMPERATURE.....	38
3.3.1.2. CALCULATION OF AIR VELOCITY IN GLAZING CAVITY	41
3.3.1.2.1. Thermally Driven Ventilation	41
3.3.1.2.2. Forced Ventilation.....	46
3.3.2. System of Equations for Glazing Systems Incorporating Shading Devices	46
3.3.3. Actual Cavity Width Convection Model:	48

3.3.4. Scalar Convection	49
3.3.4.1. “No SD” Case	49
3.3.4.2. “Closed SD” Case	50
3.3.5. Calculation of U – factor and SHGC	52
4. ISO/EN 10077-1 ALGORITHMS	52
4.1. Specular Glazing Layers (EN673/ISO10292).....	53
4.1.1. Definition of Outdoor and Indoor Heat Transfer Coefficients	53
4.2. Calculation of Glazing Cavity Thermal Conductance	53
4.2.1. Gas Conductance	53
4.2.2. Radiation Conductance.....	56
4.3. Calculation of U – factor	56

1. EXECUTIVE SUMMARY

This document presents mathematical models for calculating thermal performance of glazing systems in WINDOW 6 (TARCOG module). Those algorithms include calculation of glazing system thermal transmittance (U-factor), and solar heat gain coefficient (SHGC), as well as temperature distribution across the glazing system, according to ISO 15099, and ISO/EN 10077-1 standards. . Those mathematical algorithms consider glazing system as array of layers and gaps, where some layers may be in direct contact with each other (i.e., laminates). In addition, these algorithms consider shading devices and treat them as planar layers. Gas gaps, which represent space between layers, may consist of a single gas or gas mixtures.

2. INTRODUCTION

2.1. Layer

Layers, being constitutive part of glazing systems are defined with a list of geometrical and thermo-physical properties. Geometrical properties are width, height and thickness.

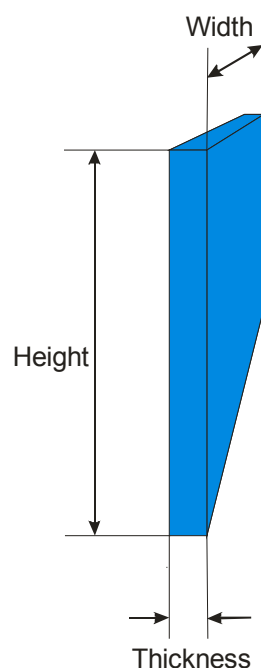


Figure 2-1: Geometry of a Layer

Geometry, shown in Figure 2-1, completely describes specular planar layer (e.g., glass, suspended film, etc.). On the other hand, layer can represent a shading device, which has several additional geometrical parameters (see Figure 2-2).

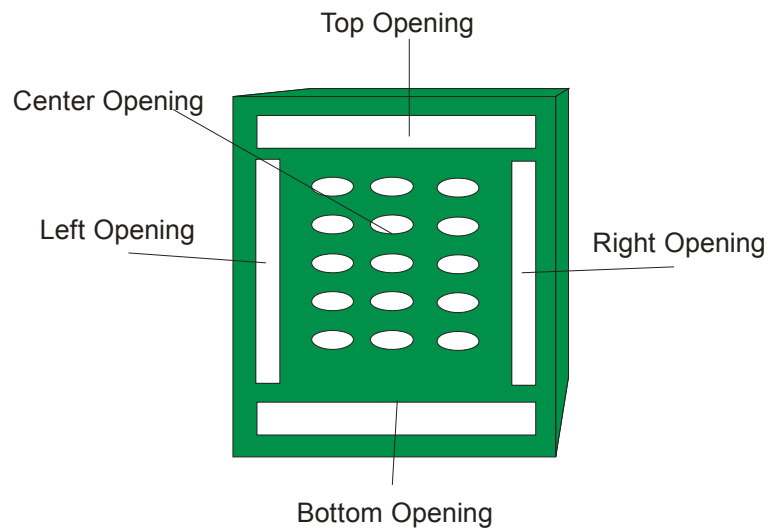


Figure 2-2: Geometry of a Generalized Shading Layer

Thermo-physical properties of a layer, which are important for the calculation of thermal and solar-optical properties of a glazing system are:

- Emissivity of both glazing surfaces,
- Transmittance as a function of wavelength
- Reflectance, back and front, as a function of wavelength
- Thermal conductivity,

Energy balance of a glazing layer or the entire glazing system is determined by the transmitted, reflected and absorbed energy, which can be calculated by the application of the conservation of energy principle. Figure 2-3 shows this energy balance:

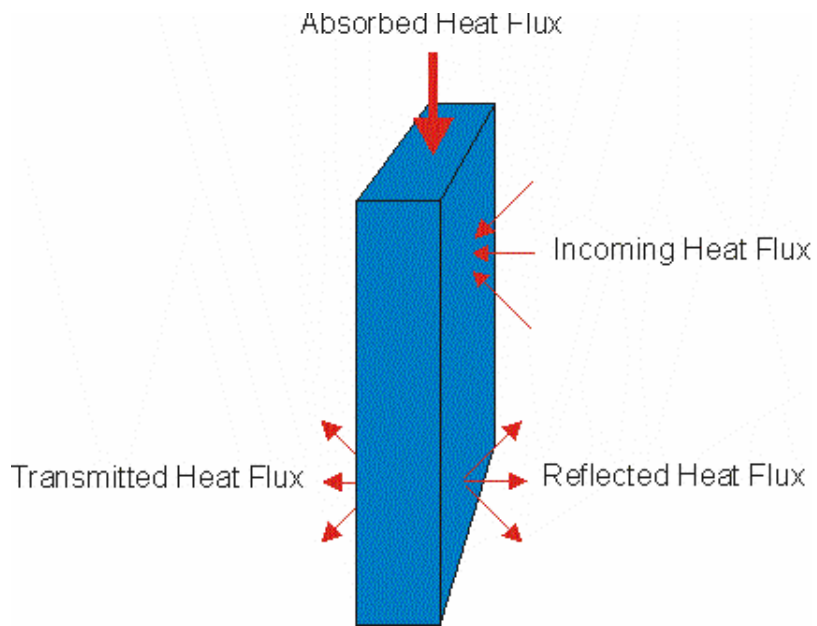


Figure 2-3: Energy Balance of a Single Glazing Layer.

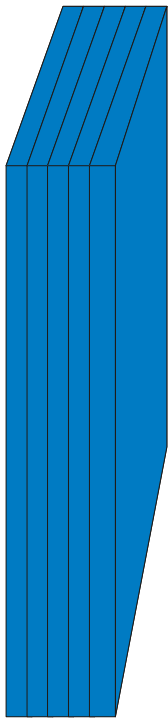


Figure 2-4: Laminate Layer

Laminate layers can be used to model through laminates, consisting of PVB interlayer(s) sandwiched between glass panes, or for the modeling of low-e coatings, or other very thin layers.

2.2. Gap

Gap consists of a single gas or gas mixture filling between two or more layers in a glazing system. Figure xxx shows gap between a double glazing.

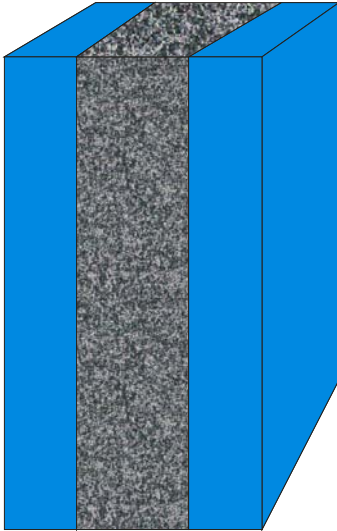


Figure 2-5: Gap (Gas Fill) Between Two Layers

Gas thermo-physical properties are:

- Thermal conductivity [$\text{W}/(\text{m}\cdot\text{K})$]
- Dynamic viscosity [$\text{g}/(\text{m}\cdot\text{s})$]
- Density [kg/m^3]
- Specific heat [$\text{J}/(\text{g}\cdot\text{K})$]

2.3. Glazing System

Glazing system, as shown in **Figure 2-6**, represents construction that consists of layer(s), separated by glazing gaps. Most of mathematical models described in this document are related to Glazing System.

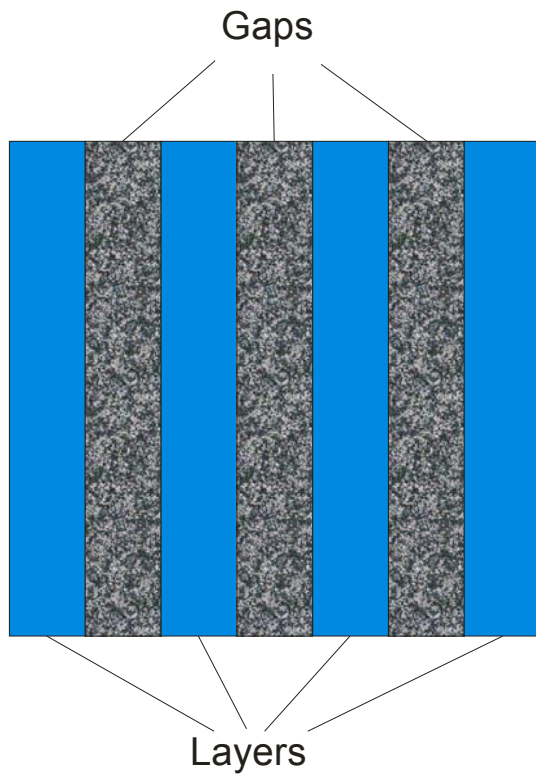


Figure 2-6: Glazing System Consisting of Specular Layers

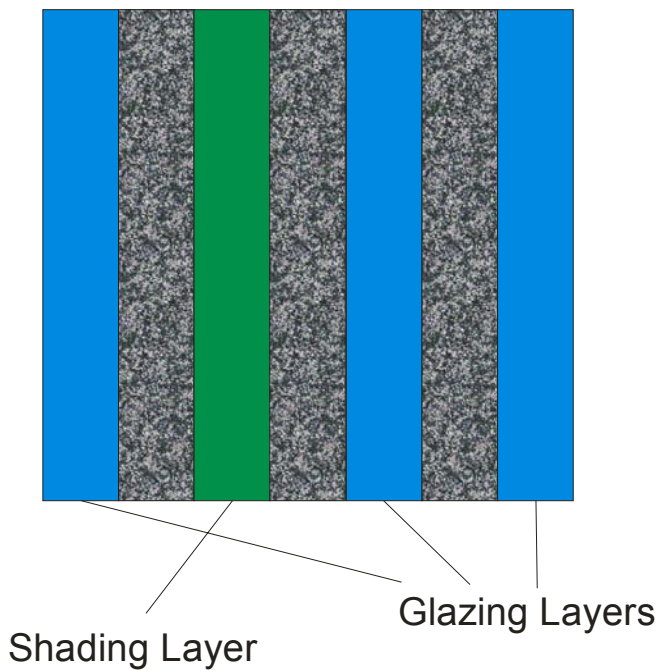


Figure 2-7: Glazing System Consisting of Specular and a Shading Layer Inside the Gap

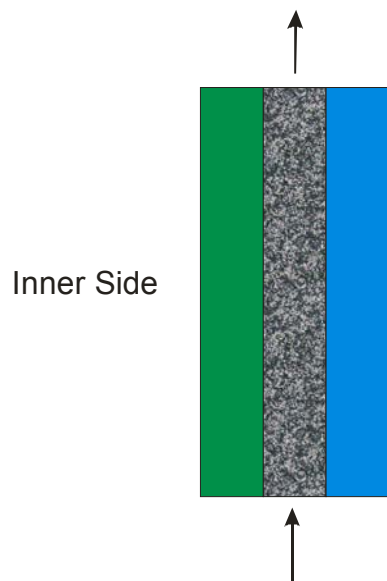


Figure 2-8: Shading Layer on Inner Side

Current implementation of the thermal model provides for large number of specular layers (1000), but only one shading layer can be present in a glazing system.

Normally, there is no physical contact between gap gasses or gap gas and indoor or outdoor environment. However, when there is a shading layer present gaps can be interconnected and indoor and outdoor environment can be connected to one of gaps (see Figure 2-9). This is referred to as ventilated gaps. In addition, the shading device layer can be porous itself.

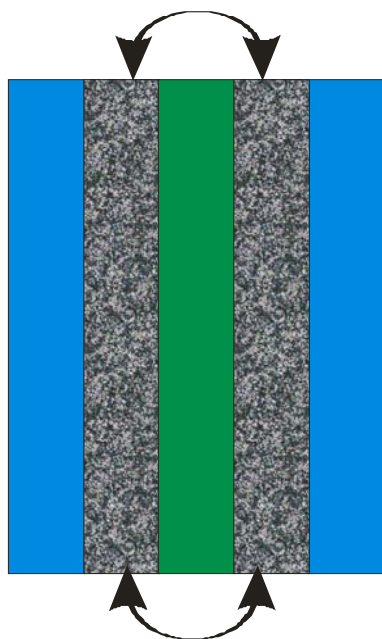


Figure 2-9: Two Gaps Connected

There can be two different types of flow in the case of ventilated gaps, a) *natural convection*, which is temperature driven due to the differences in density of the gas (i.e., buoyancy); and b) *forced convection*, caused by some external force, like a fan or wind.

3. ISO 15099 ALGORITHMS

ISO 15099 algorithms incorporate calculations for glazing systems consisting of monolithic panes, laminated panes and shading devices. The shading devices are treated in a similar way as monolithic panes (i.e. they form single layer in a glazing system), except that some of the heat transfer correlations and modeling assumptions may be somewhat different. Laminated layers are treated as monolithic layers consisting of an arbitrary number of “slices”.

3.1. GLAZING SYSTEMS CONSISTING OF MONOLITHIC LAYERS

Calculation of glazing system thermal properties is based on a comprehensive heat transfer model, with analysis of coupled conductive, convective and radiative heat transfer. Radiative heat exchange between glazing layers, as well as conductive heat transfer within each layer, can be described using first principles calculation. Convection heat transfer is modeled using heat transfer correlations, which are based on experimental measurements and numerical modeling of selected heat transfer cases (e.g., natural convection over flat plate, natural convection in rectangular enclosure, forced convection over flat plate, etc.).

3.1.1. Definitions

Before the presentation of the algorithms and mathematical models, some definitions are necessary.

- Orientation of the glazing system: Outdoor (exterior) environment is always located on the left, while indoor (interior) side is always located on the right side of the glazing system (see **Figure 3-1**)
- Layer and gap numbering is done from left to right (see Figure 6.2).
- Each layer has front and back surface, labeled “f” and “b” (see **Figure 3-3**)
- Glazing system properties also have front and back side (see **Figure 3-4**)

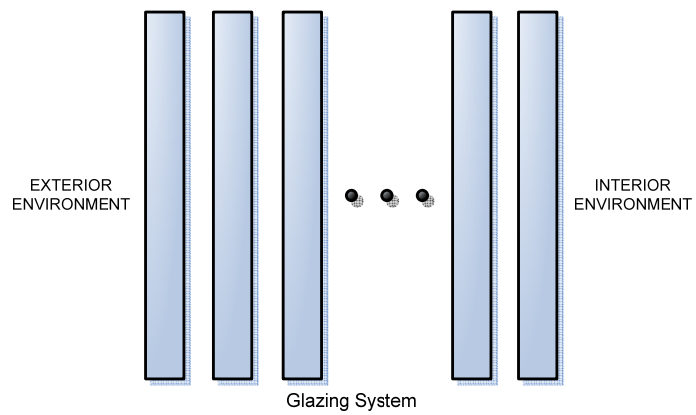


Figure 3-1: Orientation of the Glazing System with Respect to Outdoor (Exterior) and Indoor (Interior) Environment

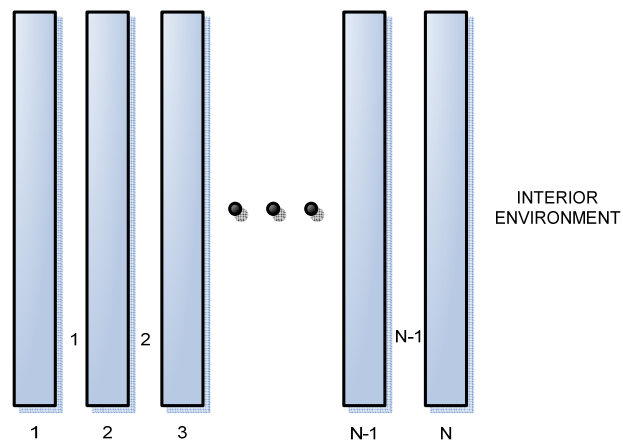


Figure 3-2: Layer and Gap Numbering

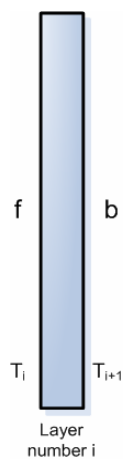


Figure 3-3: Layer Number i

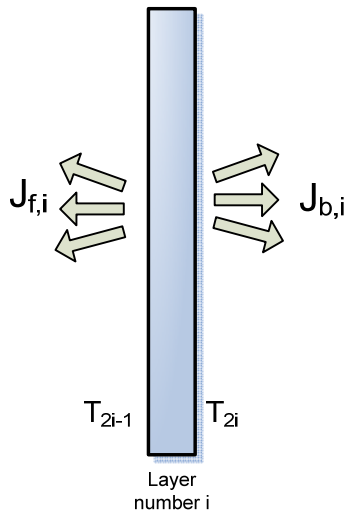


Figure 3-4: Temperatures and Energy Balance Notation

3.1.2. Heat Transfer Calculations

Figure 3-5 shows the glazing system, consisting of n layers, that is subjected to the set of standard boundary conditions. Each glazing layer is described with three longwave infra-red (IR) optical properties – the front and back surface emissivities, $\varepsilon_{f,i}$ and $\varepsilon_{b,i}$, and the transmittance τ_i .

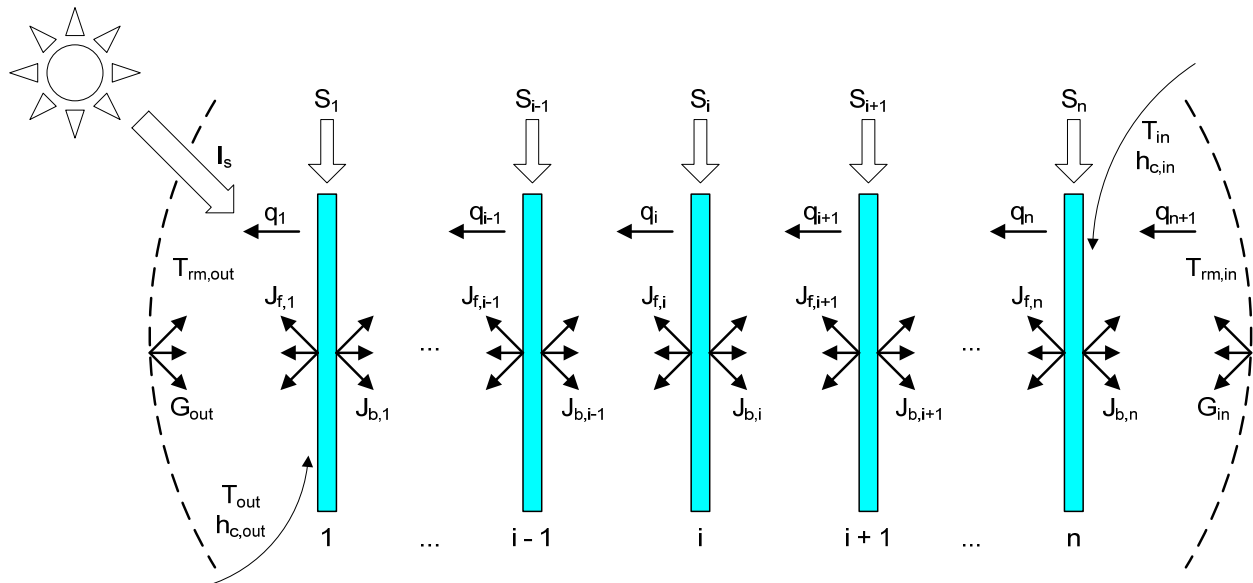


Figure 3-5: Numbering System, Boundary Conditions and Energy Balance for N-Layer Glazing System

3.1.2.1. ENERGY BALANCE

For each layer in the glazing system, shown in **Figure 3-5**, energy balance is set up, and the values of four variables are sought. These are the temperatures of the outdoor and indoor facing surfaces, $T_{f,i}$ and $T_{b,i}$, plus the radiant heat fluxes leaving the front and back facing surfaces (i.e. the radiosities), $J_{f,i}$ and $J_{b,i}$. In terms of these variables the heat flux across the i^{th} gap (i.e. q_i) is:

$$q_i = h_{c,i} [T_{f,i} - T_{b,i-1}] + J_{f,i} - J_{b,i-1} \quad [3.1-1]$$

Similarly, the heat flux across $(i+1)^{\text{th}}$ gap is determined as:

$$q_{i+1} = h_{c,i+1} [T_{f,i+1} - T_{b,i}] + J_{f,i+1} - J_{b,i} \quad [3.1-2]$$

The solution (i.e. temperatures at each glazing surface and corresponding radiant fluxes) is generated by applying the following four equations at each layer:

$$q_i = S_i + q_{i+1} \quad [3.1-3]$$

$$J_{f,i} = \epsilon_{f,i} \sigma T_{f,i}^4 + \tau_i J_{f,i+1} + \rho_{f,i} J_{b,i-1} \quad [3.1-4]$$

$$J_{b,i} = \epsilon_{b,i} \sigma T_{b,i}^4 + \tau_i J_{b,i-1} + \rho_{b,i} J_{f,i+1} \quad [3.1-5]$$

$$T_{b,i} - T_{f,i} = \frac{t_{g,i}}{2k_{g,i}} [2q_{i+1} + S_i] \quad [3.1-6]$$

Equation [3.1-3] describes an energy balance imposed at the surfaces of the i^{th} glazing layer. Equations [3.1-4] and [3.1-5] define the radiosities at the i^{th} glazing, where $\rho_{f,i} = 1 - \epsilon_{f,i} - \tau_i$ and $\rho_{b,i} = 1 - \epsilon_{b,i} - \tau_i$, while the temperature difference across the i^{th} glazing layer is given by equation [3.1-6].

In all, $4n$ (n – number of glazing layers) equations can be written on the glazing system. The equations contain terms in temperature ($T_{f,i}, T_{b,i}$) and black emissive power

($E_{bf,i} = \sigma T_{f,i}^4, E_{bb,i} = \sigma T_{b,i}^4$) and, hence, are nonlinear. They would become linear only if solved in terms of black emissive power instead of temperature (Note: The system of equations is still non-linear due to fourth power of temperature in radiation terms and $1/3^{\text{rd}}$ and $1/4^{\text{th}}$ power of temperature in natural convection terms and therefore needs to be solved iteratively, but the appearance of equations for single iteration is linear and for given temperature field allows solution of linear system of equations). Therefore is necessary to define two new quantities:

convection heat transfer coefficient based on emissive power

$$\hat{h}_i = h_{c,i} \frac{T_{f,i} - T_{b,i-1}}{E_{bf,i} - E_{bb,i-1}} \quad [3.1-7]$$

conduction heat transfer coefficient based on emissive power

$$\hat{h}_i^{gl} = \frac{k_{gl,i}}{t_{gl,i}} \cdot \frac{T_{b,i} - T_{f,i}}{E_{bb,i} - E_{bf,i}} \quad [3.1-8]$$

Application of black emissive power terms (i.e. $E_{bf,i}$ and $E_{bb,i}$) and heat transfer coefficients based on emissive power (i.e. \hat{h}_i and \hat{h}_i^{gl}) gives the following relations for heat fluxes across gas spaces:

$$q_i = \hat{h}_i [E_{bf,i} - E_{bb,i-1}] + J_{f,i} - J_{b,i-1} \quad [3.1-9]$$

$$q_{i+1} = \hat{h}_{i+1} [E_{bf,i+1} - E_{bb,i}] + J_{f,i+1} - J_{b,i} \quad [3.1-10]$$

The basic energy balance equations [3.1-3] – [3.1-6] are transformed into the following system:

$$q_i = S_i + q_{i+1} \quad [3.1-11]$$

$$J_{f,i} = \epsilon_{f,i} E_{bf,i} + \tau_i J_{f,i+1} + \rho_{f,i} J_{b,i-1} \quad [3.1-12]$$

$$J_{b,i} = \epsilon_{b,i} E_{bb,i} + \tau_i J_{b,i-1} + \rho_{b,i} J_{f,i+1} \quad [3.1-13]$$

$$\hat{h}_i^{gl} [E_{bb,i} - E_{bf,i}] = 0.5S_i + \hat{h}_{i+1} [E_{bf,i+1} - E_{bb,i}] + J_{f,i+1} - J_{b,i} \quad [3.1-14]$$

This system of 4n non-linear equations can be solved using iterative solution algorithm that is comprised of following steps:

1. Calculation of initial glazing layer temperatures
2. Calculation of heat transfer coefficients based on temperatures defined in previous step
3. Solution of the system of linear equations and definition of new sets of temperatures at each glazing layer
4. Convergence checking (comparison of new sets of temperatures to old sets)
If each temperature in the new set is not equal to the corresponding temperature in the old set within defined tolerance, the new sets are used to replace the old sets and the calculation proceeds to the second step.

This calculation procedure is described in more detail in Section 3.1.2.4.

3.1.2.2. INITIAL TEMPERATURE DISTRIBUTION

Initial glazing layer temperatures are calculated assuming a constant temperature gradient across the window. Thus, temperature at each glazing layer surface can be determined by the following equation:

$$T_i = T_{out} + x_i \frac{T_{in} - T_{out}}{t_{gs}} \quad [3.1-15]$$

where,

x_i – distance between i^{th} glazing layer and outdoor environment

t_{gs} – thickness of whole glazing system

3.1.2.3. BOUNDARY CONDITIONS

3.1.2.3.1. Outdoor Heat Transfer Coefficients

Outdoor radiation heat transfer coefficient ($h_{r,out}$) is calculated using following two relations:

$$h_{r,out} = 4\sigma\varepsilon_{f,1}\left(\frac{T_{rm,out} + T_{f,1}}{2}\right)^3 \quad [3.1-16]$$

$$h_{r,out} = \frac{G_{out} - R_{f,1}}{T_{rm,out} - T_{f,1}} \quad [3.1-17]$$

where,

$\varepsilon_{f,1}$ – emissivity of the front surface of the first glazing layer

$T_{f,i}$ – temperature of the front surface of the first glazing layer

$R_{f,i}$ – radiative flux leaving the front surface of the first glazing layer

In the first iteration, $h_{r,out}$ is calculated using equation [3.1–16], but in the second and all later iterations (if necessary), equation [3.1–17] is used.

Outdoor convection heat transfer coefficient ($h_{c,out}$) depends on method for defining outdoor combined heat transfer coefficient (h_{out}).

If the value of h_{out} , which incorporates effects of both convective and radiative heat transfer, is prescribed, outdoor convection heat transfer coefficient is calculated as:

$$h_{c,out} = h_{out} - h_{r,out} \quad [3.1-18]$$

Otherwise, calculation of $h_{c,out}$ is based on the known value for outdoor wind speed:

$$h_{c,out} = 4 + 4w_s \quad [3.1-19]$$

where,

w_s – outdoor wind speed near glass surface,

Combined outdoor surface heat transfer coefficient (h_{out}) is:

$$h_{out} = h_{c,out} + h_{r,out} \quad [3.1-20]$$

When the outdoor convection heat transfer coefficient ($h_{c,out}$) is determined, corresponding *outdoor heat transfer coefficient based on emissive power* can be defined as:

$$\hat{h}^{out} = h_{c,out} \frac{T_{f,1} - T_{amb}}{E_{bf,1} - G_{out}} \quad [3.1-21]$$

where,

$E_{bf,1}$ – emissive power of the front surface of the first glazing layer

T_{amb} – outdoor environment temperature, given as:

$$T_{amb} = \frac{h_{c,out} T_{out} + h_{r,out} T_{rm,out}}{h_{c,out} + h_{r,out}} \quad [3.1-22]$$

3.1.2.3.2. Indoor Heat Transfer Coefficients

Indoor radiation heat transfer coefficient ($h_{r,in}$) is calculated using different relations for the first and for all other iterations. In the first iteration, calculation is performed according to equation [3.1-23], while equation [3.1-24] gives relation used in all other iterations.

$$h_{r,in} = 4\sigma\varepsilon_{b,n} \left(\frac{T_{rm,in} + T_{f,n}}{2} \right)^3 \quad [3.1-23]$$

$$h_{r,out} = \frac{G_{in} - R_{b,n}}{T_{rm,in} - T_{b,n}} \quad [3.1-24]$$

where,

$\varepsilon_{b,n}$ – emissivity of the back surface of the n^{th} glazing layer

$T_{b,n}$ – temperature of the back surface of the n^{th} glazing layer

$R_{b,n}$ – radiative flux leaving the back surface of the n^{th} glazing layer

Indoor convection heat transfer coefficient ($h_{c,in}$) can be determined in two ways, depending on method for calculation of indoor combined heat transfer coefficient (h_{in}). When the value of h_{in} is prescribed, indoor convection heat transfer coefficient is calculated as:

$$h_{c,in} = h_{in} - h_{r,in} \quad [3.1-25]$$

Otherwise, natural convection is assumed to be on the indoor side of fenestration system, and $h_{c,in}$ can be determined as:

$$h_{c,in} = N_u \frac{k}{H} \quad [3.1-26]$$

where,

N_u – Nusselt number

k – thermal conductivity of air

H – height of the fenestration system

Nusselt number is a function of Rayleigh number, based on the height of the fenestration system, and tilt angle. Dependence on the window tilt angle is given through the following set of equations, and each of them corresponds to one particular range of tilt angle:

Windows inclined from 0° to 15° ($0^\circ \leq \theta < 15^\circ$)

$$N_u = 0.13 Ra_H^{1/3} \quad [3.1-27]$$

Windows inclined from 15° to 90° ($15^\circ \leq \theta \leq 90^\circ$)

$$Ra_C = 2.5 \cdot 10^5 \left(\frac{e^{0.72 \cdot \theta}}{\sin \theta} \right)^{1/5} ; \theta \text{ in degrees} \quad [3.1-28]$$

$$N_u = 0.56 \cdot (Ra_H \sin \theta)^{1/4} ; Ra_H \leq Ra_C \quad [3.1-29]$$

$$N_u = 0.13 \cdot (Ra_H^{1/3} - Ra_C^{1/3}) + 0.56 \cdot (Ra_C \sin \theta)^{1/4} ; Ra_H > Ra_C \quad [3.1-30]$$

Windows inclined from 90° to 179° ($90^\circ < \theta \leq 179^\circ$)

$$N_u = 0.56 \cdot (Ra_H \sin \theta)^{1/4} ; 10^5 \leq Ra_H \sin \theta < 10^{11} \quad [3.1-31]$$

Windows inclined from 179° to 180° ($179^\circ < \theta \leq 180^\circ$)

$$N_u = 0.58 Ra_H^{1/5} ; Ra_H \leq 10^{11} \quad [3.1-32]$$

where,

Ra_H – Rayleigh number based on the height of the fenestration system, defined as:

$$Ra_H = \frac{\rho^2 H^3 g C_p (T_{in} - T_{b,n})}{T_{mf} \mu k} \quad [3.1-33]$$

Air properties (i.e. density, specific heat, viscosity and thermal conductivity) are evaluated at the mean film temperature:

$$T_{mf} = T_{in} + \frac{1}{4} (T_{b,n} - T_{in}) \quad [3.1-34]$$

Indoor combined surface heat transfer coefficient is determined as:

$$h_{in} = h_{c,in} + h_{r,in} \quad [3.1-35]$$

After determination of the indoor convection heat transfer coefficient ($h_{c,in}$), corresponding *indoor heat transfer coefficient based on emissive power* can be found as:

$$h_{hat}^{in} = h_{c,in} \frac{T_{room} - T_{b,n}}{G_{out} - E_{bb,n}} \quad [3.1-36]$$

where,

$E_{bb,n}$ – emissive power of the back surface of the n^{th} glazing layer

T_{room} – indoor environment temperature, given as:

$$T_{room} = \frac{h_{c,in} T_{in} + h_{r,in} T_{rm,in}}{h_{c,in} + h_{r,in}} \quad [3.1-37]$$

3.1.2.3.3. Glazing Cavity Heat Transfer

Glazing cavity convective heat transfer coefficient is determined using following relation:

$$h_{c,i} = N_{u,i} \frac{k_{g,i}}{d_{g,i}} \quad [3.1-38]$$

where,

$N_{u,i}$ – Nusselt number

$k_{g,i}$ – thermal conductivity of the fill gas in the cavity

$d_{g,i}$ – thickness of the glazing cavity

Nusselt number, calculated using correlations based on experimental measurements of heat transfer across inclined air layers, is a function of the Rayleigh number, the cavity aspect ratio and the glazing system tilt angle.

The Rayleigh number can be expressed as (omitting the “i” and “g” subscripts for convenience):

$$R_a = \frac{\rho^2 d^3 g C_p (T_{f,i} - T_{b,i-1})}{T_m \mu k} \quad [3.1-39]$$

All gas fill properties (i.e. density, specific heat, viscosity and thermal conductivity) are evaluated at mean gas fill temperature, defined as:

$$T_m = \frac{T_{f,i} + T_{b,i-1}}{2} \quad [3.1-40]$$

The aspect ratio of the glazing cavity is:

$$A_{g,i} = \frac{H}{d_{g,i}} \quad [3.1-41]$$

where,

H – distance between the top and bottom of glazing cavity, usually the same as the height of the window view area

Correlation between the Nusselt number and glazing system tilt angle is given in following equations for different tilt angle ranges:

Windows inclined from 0° to 60° ($0^\circ \leq \theta < 60^\circ$)

$$N_{u,i} = 1 + 1.44 \left(1 - \frac{1708}{R_a \cos \theta} \right)^* \left[1 - \frac{1708 \sin^{1.6}(1.8 \cdot \theta)}{R_a \cos \theta} \right] + \left[\left(\frac{R_a \cos \theta}{5830} \right)^{1/3} - 1 \right]^* , R_a < 10^5 \text{ and } A_{g,i} > 20 \quad [3.1-42]$$

$$(X)^* = \frac{X + |X|}{2} \quad [3.1-43]$$

Windows inclined at 60° ($\theta = 60^\circ$)

$$N_{u,i} = (N_{u1}, N_{u2})_{\max} \quad [3.1-44]$$

$$N_{u1} = \left[1 + \left(\frac{0.0936 R_a^{0.314}}{1 + G} \right)^7 \right]^{1/7} \quad [3.1-45]$$

$$N_{u2} = \left(0.104 + \frac{0.175}{A_{g,i}} \right) R_a^{0.283} \quad [3.1-46]$$

$$G = \frac{0.5}{\left[1 + \left(\frac{R_a}{3160} \right)^{20.6} \right]^{0.1}} \quad [3.1-47]$$

Windows inclined from 60° to 90° ($60^\circ < \theta < 90^\circ$)

In this case, Nusselt number is calculated using straight-line interpolation between the results of equations [3.1-44] and [3.1-48]. These equations are valid in the ranges of $10^2 < R_a < 2 \cdot 10^7$ and $5 < A_{g,i} < 100$.

Windows inclined at 90° ($\theta = 90^\circ$)

$$N_{u,i} = (N_{u1}, N_{u2})_{\max} \quad [3.1-48]$$

where,

$$N_{u1} = 0.0673838 R_a^{1/3} ; 5 \cdot 10^4 < R_a \quad [3.1-49]$$

$$N_{u1} = 0.028154 R_a^{0.4134} ; 10^4 < R_a \leq 5 \cdot 10^4 \quad [3.1-50]$$

$$N_{u1} = 1 + 1.7596678 \cdot 10^{-10} R_a^{2.2984755} ; R_a \leq 10^4 \quad [3.1-51]$$

$$N_{u2} = 0.242 \cdot \left(\frac{R_a}{A_{g,i}} \right)^{0.272} \quad [3.1-52]$$

Windows inclined from 90° to 180° ($90^\circ < \theta < 180^\circ$)

$$N_{u,i} = 1 + (N_{uv} - 1)\sin\theta \quad [3.1-53]$$

where,

N_{uv} – Nusselt number for a vertical cavity, given by equation [3.1-48]

When the convective heat transfer coefficient is found, corresponding coefficient based on emissive power can be calculated, as described in equation [3.1-54]:

$$h_{hat,i} = h_{c,i} \frac{T_{f,i} - T_{b,i-1}}{E_{bf,i} - E_{bb,i-1}} \quad [3.1-54]$$

where,

$E_{bb,i-1}$, $E_{bf,i}$ – emissive powers of glazing surfaces surrounding the glazing cavity

3.1.2.3.4. Thermo-physical Properties of Gases

The density of individual gasses is calculated using the perfect gas law, while the other properties are determined as a linear function of mean temperature – T_m . The properties of gas mixtures are determined as per following procedure:

Density

$$\rho = \frac{P\hat{M}_{mix}}{\Re T_m} \quad [3.1-55]$$

where,

P – normal pressure (101325 Pa)

M_{mix} – molecular mass of the gas mixture, given in equation [3.1-56]

R – universal gas constant (8314.41 J/kmol)

T_m – mean gas mixture temperature, defined as per [3.1-40]

Molecular Mass

$$\hat{M}_{mix} = \sum_{i=1}^v x_i \hat{M}_i \quad [3.1-56]$$

where,

x_i – mole fraction of the i^{th} gas component in a mixture of v gases

Specific Heat

$$C_{p\,mix} = \frac{\hat{C}_{p\,mix}}{\hat{M}_{mix}} \quad [3.1-57]$$

where,

$$\hat{C}_{p\,mix} = \sum_{i=1}^u x_i \hat{C}_{p,i} \quad [3.1-58]$$

and, the molar specific heat of the i^{th} gas is:

$$\hat{C}_{p,i} = C_{p,i} \hat{M}_i \quad [3.1-59]$$

Dynamic Viscosity

$$\mu_{mix} = \sum_{i=1}^u \frac{\mu_i}{\left\{ 1 + \sum_{\substack{j=1 \\ j \neq i}}^u \varphi^{\mu}_{ij} \frac{x_j}{x_i} \right\}} \quad [3.1-60]$$

where,

$$\varphi^{\mu}_{ij} = \frac{\left[1 + \left(\frac{\mu_i}{\mu_j} \right)^{1/2} \left(\frac{\hat{M}_j}{\hat{M}_i} \right)^{1/4} \right]^2}{2\sqrt{2} \left[1 + \left(\frac{\hat{M}_i}{\hat{M}_j} \right) \right]^{1/2}} \quad [3.1-61]$$

Thermal Conductivity

$$\lambda_{mix} = \lambda'_{mix} + \lambda''_{mix} \quad [3.1-62]$$

where,

λ' – monatomic thermal conductivity

λ'' – accounts for additional energy moved by the diffusional transport of indoor energy in polyatomic gases.

$$\lambda'_{mix} = \sum_{i=1}^u \frac{\lambda'_i}{\left\{ 1 + \sum_{\substack{j=1 \\ j \neq i}}^u \psi_{ij} \frac{x_j}{x_i} \right\}} \quad [3.1-63]$$

$$\psi_{ij} = \frac{\left[1 + \left(\frac{\lambda'_i}{\lambda'_j} \right)^{1/2} \left(\frac{\hat{M}_i}{\hat{M}_j} \right)^{1/4} \right]^2}{2\sqrt{2} \left[1 + \left(\frac{\hat{M}_i}{\hat{M}_j} \right) \right]^{1/2}} \cdot \left[1 + 2,41 \frac{(\hat{M}_i - \hat{M}_j)(\hat{M}_i - 0,142\hat{M}_j)}{(\hat{M}_i + \hat{M}_j)^2} \right] \quad [3.1-64]$$

$$\lambda''_{mix} = \sum_{i=1}^u \frac{\lambda''_i}{\left\{ 1 + \sum_{\substack{j=1 \\ j \neq i}}^u \phi^{\lambda}_{ij} \frac{x_j}{x_i} \right\}} \quad [3.1-65]$$

where the expression for $\phi_{i,j}$ is given as:

$$\phi^{\lambda}_{ij} = \frac{\left[1 + \left(\frac{\lambda'_i}{\lambda'_j} \right)^{1/2} \left(\frac{\hat{M}_i}{\hat{M}_j} \right)^{1/4} \right]^2}{2\sqrt{2} \left[1 + \left(\frac{\hat{M}_i}{\hat{M}_j} \right) \right]^{1/2}} \quad [3.1-66]$$

To find λ_{mix} , it is necessary to:

1. calculate λ'_i

$$\lambda'_i = \frac{15}{4} \frac{\Re}{\hat{M}_i} \mu_i \quad [3.1-67]$$

2. calculate λ''_i

$$\lambda''_i = \lambda_i - \lambda'_i \quad [3.1-68]$$

λ_i – conductivity of the i^{th} fill gas component

3. use λ'_i to calculate λ'_{mix}

4. use λ''_i to calculate λ''_{mix}

5. determine λ_{mix} as per equation [3.1-62]

3.1.2.3.5. Interaction with the environment

The effect of boundary conditions imposed by the environment on the glazing system is given by:

Temperatures

Outdoor and indoor temperatures are defined as:

$$T_{b,0} = T_{out} \quad [3.1-69]$$

$$T_{f,n+1} = T_{in} \quad [3.1-70]$$

where,

T_{out} – outdoor air temperature

T_{in} – indoor air temperature

Long Wave Irradiance

Outdoor irradiance is set as:

$$J_{b,0} = G_{out} = \sigma T_{rm,out}^4 \quad [3.1-71]$$

where,

$T_{rm,out}$ – outdoor mean radiant temperature, calculated as:

$$T_{rm,out} = \left\{ \frac{[F_{gd} + (1 - f_{clr})F_{sky}]\sigma T_{out}^4 + f_{clr}F_{sky}J_{sky}}{\sigma} \right\}^{1/4} \quad [3.1-72]$$

f_{clr} – fraction of the sky that is clear

J_{sky} – radiosity of the clear sky, defined as:

$$J_{sky} = 5.31 \cdot 10^{-13} T_{out}^6 \quad [3.1-73]$$

F_{sky} – view factor from the outdoor surfaces of the fenestration system to the sky, defined as:

$$F_{sky} = \frac{1 + \cos \theta}{2} \quad [3.1-74]$$

θ – glazing system tilt angle measured from horizontal

F_{gd} – view factor from the outdoor surfaces of the fenestration system to the ground, defined as:

$$F_{gd} = 1 - F_{sky} \quad [3.1-75]$$

Indoor irradiance is:

$$J_{f,n+1} = G_{in} = \sigma T_{rm,in}^4 \quad [3.1-76]$$

where,

$T_{rm,in}$ – indoor mean radiant temperature

Indoor mean radiant temperature is usually assumed to be equal to the indoor air temperature, thus indoor irradiance becomes:

$$G_{in} = \sigma T_{in}^4 \quad [3.1-77]$$

Convection

Convection at the outdoor and indoor glazing surfaces is defined as:

$$h_{c,1} = h_{c,out} \quad [3.1-78]$$

$$h_{c,n+1} = h_{c,in} \quad [3.1-79]$$

where,

$h_{c,out}$ – outdoor convective heat transfer coefficient

$h_{c,in}$ – indoor convective heat transfer coefficient

3.1.2.4. SOLUTION OF THE SYSTEM OF NON-LINEAR EQUATIONS

The system of basic energy balance equations for each glazing layer, expressed in terms of black emissive power in equations [3.1-11] – [3.1-14], is solved as follows.

Using equations [3.1-9] – [3.1-14], which describe heat fluxes across gas spaces and glass layers, as well as boundary conditions in section 3.1.2.3.5, the following system of non-linear equations is obtained:

$$J_{f,1} + \hat{h}^{out} E_{bf,1} + \hat{h}_2 E_{bb,1} + J_{b,1} - J_{f,2} - \hat{h}_2 E_{bf,2} = S_1 + G_{out} + \hat{h}^{out} G_{out} \quad [3.1-80]$$

$$-J_{f,1} + \varepsilon_{f,1} E_{bf,1} + \tau_1 J_{f,2} = -\rho_{f,1} G_{out} \quad [3.1-81]$$

$$\varepsilon_{b,1} E_{bb,1} - J_{b,1} + \rho_{b,1} J_{f,2} = -\tau_1 G_{out} \quad [3.1-82]$$

$$\hat{h}_1^{gl} E_{bf,1} + (\hat{h}_1^{gl} + \hat{h}_2) E_{bb,1} + J_{b,1} - J_{f,2} - \hat{h}_2 E_{bf,2} = 0.5 S_1 \quad [3.1-83]$$

...

$$-\hat{h}_{i,i} E_{bb,i-1} - J_{b,i-1} + J_{f,i} + \hat{h}_{i,i} E_{bf,i} + \hat{h}_{i+1} E_{bb,i} + J_{b,i} - J_{f,i+1} - \hat{h}_{i+1} E_{bf,i+1} = S_i \quad [3.1-84]$$

$$\rho_{f,i} J_{b,i-1} - J_{f,i} + \varepsilon_{f,i} E_{bf,i} + \tau_i J_{f,i+1} = 0 \quad [3.1-85]$$

$$\tau_i J_{b,i-1} + \varepsilon_{b,i} E_{bb,i} - J_{b,i} + \rho_{b,i} J_{f,i+1} = 0 \quad [3.1-86]$$

$$\hat{h}_i^{gl} E_{bf,i} + (\hat{h}_i^{gl} + \hat{h}_{i+1}) E_{bb,i} + J_{b,i} - J_{f,i+1} - \hat{h}_{i+1} E_{bf,i+1} = 0.5 S_i \quad [3.1-87]$$

...

$$-\hat{h}_n E_{bb,n-1} - J_{b,n-1} + J_{f,n} + \hat{h}_n E_{bf,n} + \hat{h}^{in} E_{bb,n} + J_{b,n} = S_n + G_{in} + \hat{h}^{in} G_{in} \quad [3.1-88]$$

$$\rho_{f,n} J_{b,n-1} - J_{f,n} + \varepsilon_{f,n} E_{bf,n} = -\tau_n G_{in} \quad [3.1-89]$$

$$\tau_n J_{b,n-1} + \varepsilon_{b,n} E_{bb,n} - J_{b,n} = -\rho_{b,n} G_{in} \quad [3.1-90]$$

$$\hat{h}_n^{gl} E_{bf,n} + (\hat{h}_n^{gl} + \hat{h}^{in}) E_{bb,n} + J_{b,n} = 0.5 S_n + G_{in} + \hat{h}^{in} G_{in} \quad [3.1-91]$$

The equations [3.1-80] – [3.1-91] can be set in a matrix form $[A] [X] = [B]$ for the whole glazing system.

Elements of matrix A are long wave optical properties of each layer (i.e. front and back surface emissivities), as well as heat transfer coefficients, based on emissive power, calculated in a previous step.

$$[A] = \begin{bmatrix} 1 & \hat{h}^{out} & \hat{h}_2 & 1 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ -1 & \varepsilon_{f,1} & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_{b,1} & -1 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & -\hat{h}_1^{gl} & \hat{h}_1^{gl} + \hat{h}_2 & 1 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 1 & \hat{h}_i & \hat{h}_{i+1} & 1 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & -1 & \varepsilon_{f,i} & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & \varepsilon_{b,i} & -1 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & -\hat{h}_i^{gl} & \hat{h}_i^{gl} + \hat{h}_{i+1} & 1 & \dots & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 1 & \hat{h}_n & \hat{h}^{in} & 1 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & -1 & \varepsilon_{f,n} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & \varepsilon_{b,n} & -1 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & -\hat{h}_n^{gl} & \hat{h}_n^{gl} + \hat{h}^{in} & 1 \end{bmatrix}$$

The column vector X consists of variables that are sought for each glazing layer (i.e. radiosities and black emissive powers).

Finally, the column vector B contains terms of absorbed solar fluxes in each glazing layer; terms of radiative energy from outdoor and indoor environment that irradiates glazing system surfaces, as well as portions of outdoor and indoor radiative energy that are reflected from 1st and nth layer and transmitted through them.

$$[X] = \begin{bmatrix} J_{f,1} \\ E_{bf,1} \\ E_{bb,1} \\ J_{b,1} \\ \dots \\ J_{f,i} \\ E_{bf,i} \\ E_{bb,i} \\ J_{b,i} \\ \dots \\ J_{f,n} \\ E_{bf,n} \\ E_{bb,n} \\ J_{b,n} \end{bmatrix} \quad [B] = \begin{bmatrix} S_1 + G_{out} + \hat{h}^{out} G_{out} \\ -\rho_{f,1} G_{out} \\ -\tau_1 G_{out} \\ 0.5 S_1 \\ \dots \\ S_i \\ 0 \\ 0 \\ 0.5 S_i \\ \dots \\ S_n + G_{in} + \hat{h}^{in} G_{in} \\ -\tau_n G_{in} \\ -\rho_{b,n} G_{in} \\ 0.5 S_n + G_{in} + \hat{h}^{in} G_{in} \end{bmatrix}$$

The solution method consists of decomposition of the matrix A, and solving matrix equation $[A] [X] = [B]$, where $[A]$ is decomposed matrix.

That way, sets of black emissive power (i.e. $E_{bf,i}$ and $E_{bb,i}$) are found and new sets of temperatures can be determined as:

$$T_{f,i} = \left(\frac{E_{bf,i}}{\sigma} \right)^{1/4} \quad \text{and} \quad T_{b,i} = \left(\frac{E_{bb,i}}{\sigma} \right)^{1/4} \quad [3.1-92]$$

where,

$$\sigma = 5.6697 \cdot 10^{-8} \frac{W}{m^2 K^4} \text{ – Stefan – Boltzmann's constant}$$

3.1.2.5. CONVERGENCE CHECKING

When the new sets of front and back layer surface temperatures are determined, they are compared to the old sets. If each member of the new set is not equal to corresponding member of the old set within the prescribed tolerance (default is 10^{-8} K), the new set is used to replace the old set and solution algorithm proceeds to the second step (i.e. calculation of heat transfer coefficients).

3.1.3. Calculation of U – factor

Calculation of U – factor is based on heat flux through the glazing system for the specified environmental conditions, which is determined as per section 3.1.2, but

without incident solar radiation (i.e. $I_s = 0$). It means that the fluxes of absorbed solar radiation in each glazing layer, are set to zero in corresponding energy balance equations (i.e. $S_i = 0$).

U-factor is determined as the reciprocal of the total glazing system thermal resistance – R_{tot} .

$$U = \frac{1}{R_{tot}} \quad [3.1-93]$$

The total thermal resistance of a glazing system can be calculated by summing the thermal resistance on the outdoor side of the glazing system, the individual thermal resistance values of the glazing layers and glazing cavities and the thermal resistance on the indoor side of the glazing system:

$$R_{tot} = R_{out} + \sum_{i=1}^n R_{gl,i} + \sum_{i=1}^n R_{gap,i} + R_{in} \quad [3.1-94]$$

The thermal resistance on the outdoor side of the glazing system – R_{out} , can be described by the following equation:

$$R_{out} = \frac{T_{f,1} - T_{amb}}{h_{c,out}(T_{f,1} - T_{out}) + J_{f,1} - G_{out}} \quad [3.1-95]$$

The thermal resistance of the glazing layer – $R_{gl,i}$, is:

$$R_{gl,i} = \frac{t_{gl,i}}{k_{gl,i}} \quad [3.1-96]$$

The thermal resistance of the glazing cavity – $R_{gap,i}$, is given as:

$$R_{gap,i} = \frac{T_{f,i} - T_{b,i-1}}{h_{c,i}(T_{f,i} - T_{b,i-1}) + J_{f,i} - J_{b,i-1}} \quad [3.1-97]$$

Finally, the thermal resistance on the indoor side of the glazing system – R_{in} , is:

$$R_{in} = \frac{T_{room} - T_{b,n}}{h_{c,in}(T_{room} - T_{b,n}) + G_{in} - J_{b,n}} \quad [3.1-98]$$

3.1.4. Calculation of Solar Heat Gain Coefficient

Opposite to the calculation of thermal transmittance (U – factor), calculation of solar heat gain coefficient (SHGC) incorporates all effects of the incident solar radiation.

The solar heat gain coefficient (SHGC) is determined by the difference between the heat fluxes into the indoor environment with and without incident solar radiation, where the both fluxes are found as per procedure given in section 3.1.2.

$$SHGC = T_{sol} + \frac{q_{in(l_s=0)} - q_{in}}{I_s} \quad [3.1-99]$$

where,

T_{sol} – total solar transmittance of the glazing system (–)

$q_{in(l_s=0)}$ – heat flux into the indoor environment without incident solar radiation (W/m²)

q_{in} – heat flux into the indoor environment with incident solar radiation (W/m²)

I_s – incident solar radiation (W/m²)

3.2. GLAZING SYSTEMS WITH LAMINATED LAYERS

For a glazing system incorporating laminated layers, the algorithm for calculation of thermal properties is very similar to the glazing systems consisting of monolithic layers, given in section 3.1. Laminated layers are modeled as monolithic layers subdivided into the arbitrary number of slices that have no gap in-between them. System of linear equations, from which layer surface temperatures and radiant fluxes (i.e. radiosities) are determined, is given here.

3.2.1. Energy Balance Equations for Laminated Layers

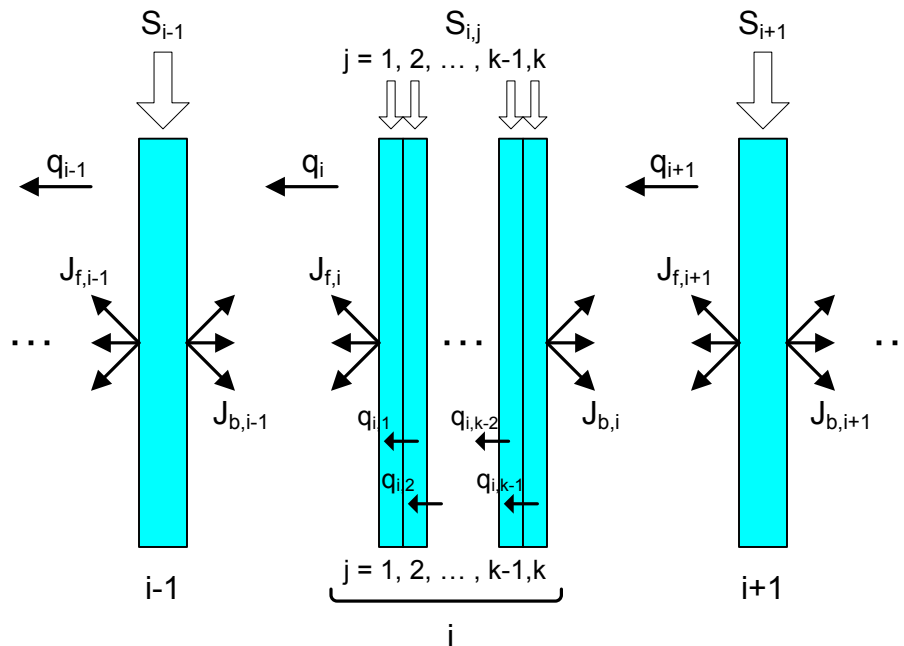


Figure 3-6: Numbering System and Energy Balance for Laminated Glazing Layers in the Glazing System

In case of laminated layer, shown in **Figure 3-6**, the basic energy balance equations [3.1-3] – [3.1-6] are modified by applying the following relations to the unexposed surfaces of the layer slices:

$$T_{b,jj} = T_{f,jj+1} \quad [3.2-1]$$

$$J_{b,jj} = J_{f,jj+1} \quad [3.2-2]$$

where,

$j = 1, \dots, k$ – number of slices in the laminated layer.

In that way, the following energy balance equations are set for the slices of the laminated glazing layer:

SLICE 1

$$q_i = S_{i,1} + q_{i,1} \quad [3.2-3]$$

$$J_{f,i} = \varepsilon_{f,i} \sigma T_{f,i}^4 + \tau_i J_{f,i+1} + \rho_{f,i} J_{b,i-1} \quad [3.2-4]$$

$$T_{b,i1} - T_{f,i} = \frac{t_{sl,i1}}{2k_{sl,i1}} (2q_{i,1} + S_{i,1}) \quad [3.2-5]$$

SLICE 2

$$q_{i,1} = S_{i,2} + q_{i,2} \quad [3.2-6]$$

$$T_{f,i2} = T_{b,i1} \quad [3.2-7]$$

$$T_{b,i2} - T_{f,i2} = \frac{t_{sl,i2}}{2k_{sl,i2}} (2q_{i,2} + S_{i,2}) \quad [3.2-8]$$

...

SLICE k-1

$$q_{i,k-2} = S_{i,k-1} + q_{i,k-1} \quad [3.2-9]$$

$$T_{f,ik-1} = T_{b,ik-2} \quad [3.2-10]$$

$$T_{b,ik-1} - T_{f,ik-1} = \frac{t_{sl,ik-1}}{2k_{sl,ik-1}} (2q_{i,k-1} + S_{i,k-1}) \quad [3.2-11]$$

SLICE k

$$q_{i,k-1} = S_{i,k} + q_{i+1} \quad [3.2-12]$$

$$T_{f,ik} = T_{b,ik-1} \quad [3.2-13]$$

$$J_{b,i} = \varepsilon_{b,i} \sigma T_{b,i}^4 + \tau_i J_{b,i-1} + \rho_{b,i} J_{f,i+1} \quad [3.2-14]$$

$$T_{b,i} - T_{f,ik} = \frac{t_{sl,ik}}{2k_{sl,ik}} (2q_{i+1} + S_{i,k}) \quad [3.2-15]$$

From the equations [3.2-3], [3.2-6], [3.2-9] and [3.2-12], which describe energy balances imposed at the surfaces of the laminated layer slices, the energy balance relation for the whole laminated layer can be derived as:

$$q_i = \sum_{j=1}^k S_{i,j} + q_{i+1} \quad [3.2-16]$$

At the same time, the equations [3.2-5], [3.2-8], [3.2-11] and [3.2-15], which define temperature difference across the laminated layer slices (this is a variation of equation [3.1-6] for monolithic layers), in conjunction with the equations [3.2-7], [3.2-10] and [3.2-13], which define temperature at the unexposed surfaces of the layer slices, give the modified relation for the temperature difference across the whole laminated layer. If we use 1st and 2nd slice of laminated layer as an example, temperature difference equation becomes:

$$T_{b,i1} - T_{f,i} = \frac{t_{sl,i1}}{2k_{sl,i1}} (2q_{i,1} + S_{i,1}) \quad [3.2-17]$$

$$T_{b,i2} - T_{f,i2} = \frac{t_{sl,i2}}{2k_{sl,i2}} (2q_{i,2} + S_{i,2}) \quad [3.2-18]$$

Since the temperature of adjacent surfaces of the 1st and 2nd slice is equal (i.e. $T_{b,i1} = T_{f,i2}$), as per assumption given in equation [3.2-1], the following relation can be derived from equations [3.2-17] and [3.2-18]:

$$T_{b,i2} - T_{f,i} = \frac{t_{sl,i1}}{2k_{sl,i1}} (2q_{i,1} + S_{i,1}) + \frac{t_{sl,i2}}{2k_{sl,i2}} (2q_{i,2} + S_{i,2}) \quad [3.2-19]$$

Application of the same methodology to all other slices gives the relation for temperature difference across the whole laminated layer:

$$T_{b,i} - T_{f,i} = \frac{t_{sl,i1}}{2k_{sl,i1}} (2q_{i,1} + S_{i,1}) + \frac{t_{sl,i2}}{2k_{sl,i2}} (2q_{i,2} + S_{i,2}) + \dots + \frac{t_{sl,ik-1}}{2k_{sl,ik-1}} (2q_{i,k-1} + S_{i,k-1}) + \frac{t_{sl,ik}}{2k_{sl,ik}} (2q_{i+1} + S_{i,k}) \quad [3.2-20]$$

From equations [3.2-3], [3.2-6], [3.2-9] and [3.2-12] it is obvious that heat fluxes entering the back surface of each slice (i.e. $q_{i,1}$, $q_{i,2}$, ..., $q_{i,k-2}$, $q_{i,k-1}$) can be expressed in terms of fluxes of solar energy absorbed in the previous slices – $S_{i,2}$, ..., $S_{i,k-1}$, $S_{i,k}$ (going toward outdoor environment), and heat flux entering the back surface of laminated layer (i.e. q_{i+1}):

$$q_{i,j} = \sum_{p=j+1}^k S_{i,p} + q_{i+1} \quad [3.2-21]$$

After using relation [3.2-21] for fluxes $q_{i,1}$, $q_{i,2}$, ..., $q_{i,k-2}$, $q_{i,k-1}$ and some rearrangements, the equation [3.2-20] becomes:

$$T_{b,i} - T_{f,i} = \sum_{j=1}^k \frac{t_{sl,ij}}{2k_{sl,ij}} S_{i,j} + \sum_{j=1}^{k-1} \left(\frac{t_{sl,ij}}{k_{sl,ij}} \sum_{p=j+1}^k S_{i,p} \right) + q_{i+1} \sum_{j=1}^k \frac{t_{sl,ij}}{k_{sl,ij}} \quad [3.2-22]$$

or,

$$T_{b,i} - T_{f,i} = A + Bq_{i+1} \quad [3.2-23]$$

where the terms A and B are given as:

$$A = \sum_{j=1}^k \left(\frac{t_{sl,ij}}{2k_{sl,ij}} S_{i,j} \right) + \sum_{j=1}^{k-1} \left(\frac{t_{sl,ij}}{2k_{sl,ij}} \sum_{p=j+1}^k S_{i,p} \right) \quad [3.2-24]$$

$$B = \sum_{j=1}^k \frac{t_{sl,ij}}{2k_{sl,ij}} \quad [3.2-25]$$

Using the equations [3.2-16] and [3.2-23], the system of basic energy balance equations [3.1-3] – [3.1-6] can be transformed into the following system for laminated layers:

$$q_i = \sum_{j=1}^k S_{i,j} + q_{i+1} \quad [3.2-26]$$

$$J_{f,i} = \varepsilon_{f,i} \sigma T_{f,i}^4 + \tau_i J_{f,i+1} + \rho_{f,i} J_{b,i-1} \quad [3.2-27]$$

$$J_{b,i} = \varepsilon_{b,i} \sigma T_{b,i}^4 + \tau_i J_{b,i-1} + \rho_{b,i} J_{f,i+1} \quad [3.2-28]$$

$$T_{b,i} - T_{f,i} = A + Bq_{i+1} \quad [3.2-29]$$

Using the same notation (e.g., A and B), the temperature difference terms for monolithic layers can be expressed as:

$$A = \frac{t_{gl,i}}{2k_{gl,i}} S_i \quad [3.2-30]$$

$$B = \frac{t_{gl,i}}{k_{gl,i}} \quad [3.2-31]$$

As in case of glazing systems consisting of monolithic layers, this system of energy balance equations for laminated layers would become linear only if solved in terms of black emissive power instead of temperature. Note that the system of equations is still non-linear due to fourth power of temperature in radiation terms and 1/3rd and 1/4th power of temperature in natural convection terms and therefore needs to be solved

iteratively, but the appearance of equations for single iteration is linear and for given temperature field allows solution of linear system of equations.

Considering that differences between glazing systems with laminated layers and glazing systems consisted of monolithic layers are reflected only in equations related to conductive heat transfer through the glazing layers, the new relationship for the conduction heat transfer coefficient based on emissive power, is introduced and shown equation [3.2–32].

$$\hat{h}_i^{gl} = \frac{1}{B} \cdot \frac{T_{b,i} - T_{f,i}}{E_{bb,i} - E_{bf,i}}. \quad [3.2-32]$$

The expression for the convection heat transfer coefficient based on emissive power, given in equation [3.1–7], remains the same here

3.2.2. System of Equations for Glazing Systems Incorporating Laminated Layers

Using the method, described at the beginning of section Error! Reference source not found., along with relations [3.2–26] – [3.2–29] and [3.2–32] for laminated layers, the following system of equations is obtained for the glazing system, which incorporates laminated layers:

$$J_{f,1} + \hat{h}^{out} E_{bf,1} + \hat{h}_{,2} E_{bb,1} + J_{b,1} - J_{f,2} - \hat{h}_{,2} E_{bf,2} = S_1 + G_{out} + \hat{h}^{out} G_{out} \quad [3.2-33]$$

$$-J_{f,1} + \varepsilon_{f,1} E_{bf,1} + \tau_1 J_{f,2} = -\rho_{f,1} G_{out} \quad [3.2-34]$$

$$\varepsilon_{b,1} E_{bb,1} - J_{b,1} + \rho_{b,1} J_{f,2} = -\tau_1 G_{out} \quad [3.2-35]$$

$$\hat{h}_1^{gl} E_{bf,1} + (\hat{h}_1^{gl} + \hat{h}_2) E_{bb,1} + J_{b,1} - J_{f,2} - \hat{h}_{,2} E_{bf,2} = 0.5 S_1 \quad [3.2-36]$$

...

$$-\hat{h}_i E_{bb,i-1} - J_{b,i-1} + J_{f,i} + \hat{h}_i E_{bf,i} + \hat{h}_{i+1} E_{bb,i} + J_{b,i} - J_{f,i+1} - \hat{h}_{i+1} E_{bf,i+1} = \sum_{j=1}^k S_{i,j} \quad [3.2-37]$$

$$\rho_{f,i} J_{b,i-1} - J_{f,i} + \varepsilon_{f,i} E_{bf,i} + \tau_i J_{f,i+1} = 0 \quad [3.2-38]$$

$$\tau_i J_{b,i-1} + \varepsilon_{b,i} E_{bb,i} - J_{b,i} + \rho_{b,i} J_{f,i+1} = 0 \quad [3.2-39]$$

$$\hat{h}_i^{gl} E_{bf,i} + (\hat{h}_i^{gl} + \hat{h}_{i+1}) E_{bb,i} + J_{b,i} - J_{f,i+1} - \hat{h}_{i+1} E_{bf,i+1} = \frac{A}{B} \quad [3.2-40]$$

...

$$-\hat{h}_n E_{bb,n-1} - J_{b,n-1} + J_{f,n} + \hat{h}_n E_{bf,n} + \hat{h}^{in} E_{bb,n} + J_{b,n} = S_n + G_{in} + \hat{h}^{in} G_{in} \quad [3.2-41]$$

$$\rho_{f,n} J_{b,n-1} - J_{f,n} + \varepsilon_{f,n} E_{bf,n} = -\tau_n G_{in} \quad [3.2-42]$$

$$\tau_n J_{b,n-1} + \varepsilon_{b,n} E_{bb,n} - J_{b,n} = -\rho_{b,n} G_{in} \quad [3.2-43]$$

$$\hat{h}_n^{gl} E_{bf,n} + (\hat{h}_n^{gl} + \hat{h}^{in}) E_{bb,n} + J_{b,n} = 0.5S_n + G_{in} + \hat{h}^{in} G_{in} \quad [3.2-44]$$

Similarly to the standard glazing systems, equations [3.2-33] – [3.2-44] are set in a matrix form $[A] [X] = [B]$ for the whole glazing system, consisting of laminated layers.

$$[A] = \begin{bmatrix} 1 & \hat{h}^{out} & \hat{h}_2 & 1 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ -1 & \varepsilon_{f,1} & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_{b,1} & -1 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & -\hat{h}_1^{gl} & \hat{h}_1^{gl} + \hat{h}_2 & 1 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 1 & \hat{h}_i & \hat{h}_{i+1} & 1 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & -1 & \varepsilon_{f,i} & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & \varepsilon_{b,i} & -1 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & -\hat{h}_i^{gl} & \hat{h}_i^{gl} + \hat{h}_{i+1} & 1 & \dots & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 1 & \hat{h}_n & \hat{h}^{in} & 1 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & -1 & \varepsilon_{f,n} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & \varepsilon_{b,n} & -1 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & -\hat{h}_n^{gl} & \hat{h}_n^{gl} + \hat{h}^{in} & 1 \end{bmatrix}$$

$$[X] = \begin{bmatrix} J_{f,1} \\ E_{bf,1} \\ E_{bb,1} \\ J_{b,1} \\ \dots \\ J_{f,i} \\ E_{bf,i} \\ E_{bb,i} \\ J_{b,i} \\ \dots \\ J_{f,n} \\ E_{bf,n} \\ E_{bb,n} \\ J_{b,n} \end{bmatrix} \quad [B] = \begin{bmatrix} S_1 + G_{out} + \hat{h}^{out} G_{out} \\ -\rho_{f,1} G_{out} \\ -\tau_1 G_{out} \\ 0.5S_1 \\ \dots \\ \Sigma S_{i,j} \\ 0 \\ 0 \\ \frac{A}{B} \\ \dots \\ S_n + G_{in} + \hat{h}^{in} G_{in} \\ -\tau_n G_{in} \\ -\rho_{b,n} G_{in} \\ 0.5S_n + G_{in} + \hat{h}^{in} G_{in} \end{bmatrix}$$

3.2.3. Determination of Glazing Surface Temperatures

The matrix equation $[A][X] = [B]$ is solved in the same manner as in previous case (i.e. glazing systems consisted of monolithic layers), and sets of black emissive power (i.e. $E_{bf,i}$ and $E_{bb,i}$) and radiant fluxes (i.e. $J_{f,i}$ and $J_{b,i}$) for exposed glazing surfaces are determined. Then, new sets of temperatures of exposed glazing surfaces (i.e. $T_{f,i}$ and $T_{b,i}$) are found, as described by the equation [3.1–92], and convergence checking (i.e. comparison between the new and old sets of glazing surface temperatures) is performed. After that, the old set is replaced by the new set, and iterative algorithm is repeated until the prescribed tolerance is satisfied.

When the final temperatures of the exposed glazing surfaces are found, the rest of the solution (i.e. temperatures of unexposed surfaces of layer slices) calculated using the following procedure:

1. Calculate the heat flux leaving the front facing surface of laminated layer (i.e. q_i) using the equation [3.1–1].
2. Calculate temperature of the back surface of the first slice (i.e. $T_{bi,1}$) using the following relation, derived from the equations [3.2–3] and [3.2–5]:

$$T_{b,i1} = T_{f,i} + \frac{t_{sl,i1}}{2k_{sl,i1}}(2q_i - S_{i,1}) \quad [3.2-45]$$

3. Calculate, in sequential manner, remaining temperatures of the unexposed surfaces of layer slices using similar relations, obtained from the equations that describe energy balances imposed at the surfaces of the laminated layer slices (i.e. 3.2–6, 3.2–9 and 3.2–12), equations that define temperature difference across the laminated layer slices (i.e. 3.2–8, 3.2–11 and 3.2–15) and equations that define temperature at the adjacent unexposed surfaces of the layer slices (i.e. 3.2–7, 3.2–10 and 3.2–13).

3.2.4. Calculation of U – factor and SHGC for Laminated Layers

U – factor of glazing systems with laminated layers is calculated using basically the same procedure as in previous case (i.e. glazing systems with monolithic layers).

It is determined as the reciprocal of the total glazing system thermal resistance – R_{tot} , and the R_{tot} is found by summing the thermal resistance on the outdoor side of the glazing system, thermal resistances of the glazing layers and glazing cavities and the thermal resistance on the indoor side of the glazing system.

The values of thermal resistance on the glazing system outdoor side, thermal resistance of glazing cavities and thermal resistance on the glazing system indoor side are determined as per equations [3.1–95], [3.1–97] and [3.1–98], respectively. Only difference from the previous case is reflected in thermal resistance of laminated layer, which can be found by summing individual thermal resistances of layer slices:

$$R_{gl,i} = \sum_{j=1}^k \frac{t_{sl,jj}}{k_{gl,jj}} \quad [3.2-46]$$

Regarding solar heat gain coefficient (SHGC), calculation is performed in the same manner as for glazing systems with monolithic layers using the equation [3.1–99].

3.3. GLAZING SYSTEMS WITH SHADING DEVICES

The ISO 15099 standard considers only layer types of shading devices, such as screens, curtains and blinds, which are located close and parallel to the glazing panes. The algorithm for calculation of thermal properties of glazing systems with shading devices differs from the algorithms for the other glazing system types, given in sections 3.1 and 3.2, due to modifications that the introduction of shading devices in the glazing system model requires. Differences, mainly related to the convective heat transfer through the glazing cavities surrounding shading device, lead to adjustments that should be made in the main thermal equations and, consequently, in the calculation procedure.

3.3.1. Modifications of Basic Energy Balance Equations

Since the shading device, shown as the i^{th} layer in the **Figure 3-7**, allows ventilation through the adjacent glazing cavities, the convective heat transfer in ventilated cavities can not be described as a simple heat transfer from one surface to another, as it was done in sections 3.1 and 3.2.

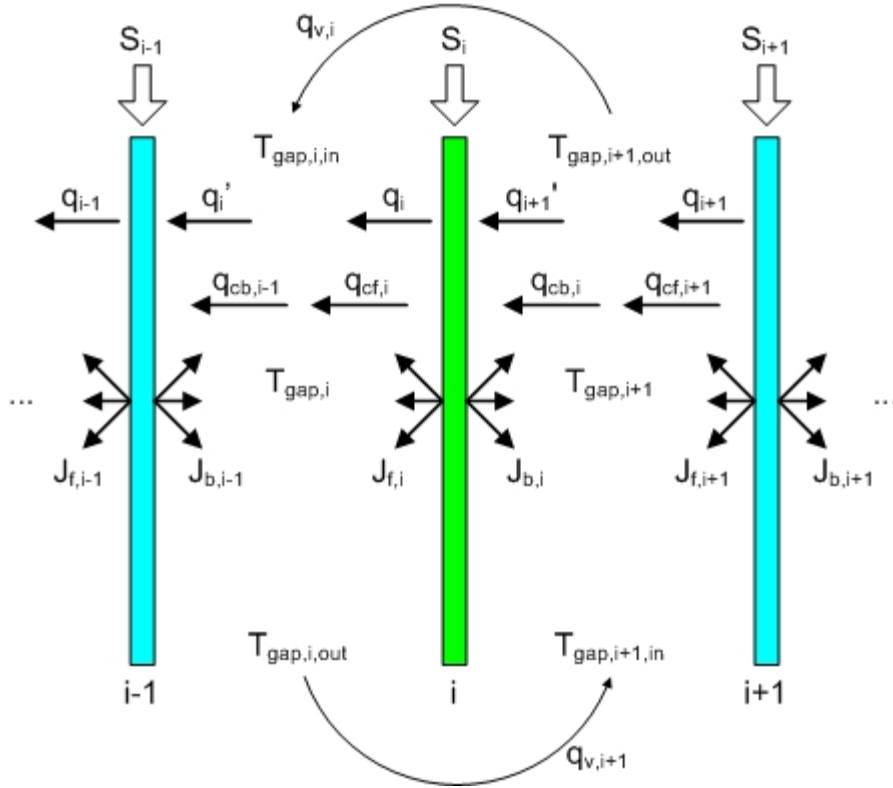


Figure 3-7: Numbering System and Energy Balance for Shading Layers in the Glazing System

In this case, convective heat flux through the ventilated cavity must be divided into two parts – convective heat flux from one layer to the gap space and convective heat flux from the gap space to another layer, where the mean gap temperature ($T_{gap,i}$) is a variable. Therefore, the convective heat flux through the i^{th} glazing cavity can be described by the following equations:

$$q_{cf,i} = h_{cv,i} (T_{f,i} - T_{gap,i}) \quad [3.3-1]$$

$$q_{cb,i-1} = h_{cv,i} (T_{gap,i} - T_{b,i-1}) \quad [3.3-2]$$

where,

$q_{cf,i}$ – convective heat flux from one layer to the gap space

$q_{cb,i-1}$ – convective heat flux from the gap space to another layer

$T_{gap,i}$ – equivalent mean temperature of the gap space (see equation 3.3–28 in the section 3.3.1.1)

$T_{f,i}$ – temperature of the surface of the i^{th} layer, facing the i^{th} glazing cavity

$T_{b,i-1}$ – temperature of the surface of the layer $i-1$, facing the i^{th} glazing cavity

$h_{cv,i}$ – convection heat transfer coefficient for ventilated cavities, given as:

$$h_{cv,i} = 2h_{cdv,i} + 4v_i \quad [3.3-3]$$

where,

$h_{cdv,i}$ – convection heat transfer coefficient for non-ventilated cavities (see section 3.1.2.3.3)

v_i – mean air velocity in the glazing cavity (see section 3.3.1.2)

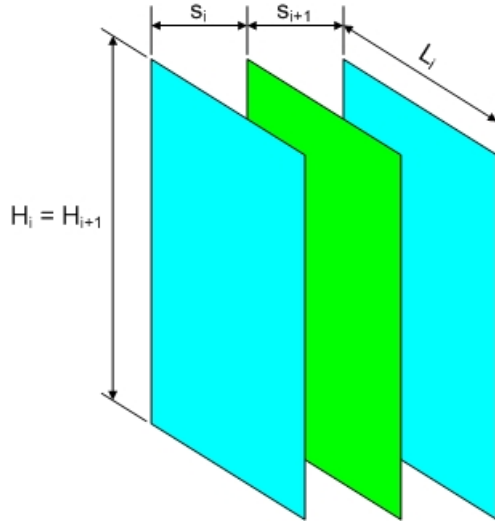


Figure 3-8: Main Dimensions of Glazing System Model with Shading Device

At the same time, the heat flux (i.e. $q_{v,i}$) supplied to or extracted from the ventilated cavity, determined as per following relation, must be included in main energy balance equations.

$$q_{v,i} = \frac{\rho_i \cdot c_{p,i} \cdot \varphi_{v,i} (T_{gap,i,in} - T_{gap,i,out})}{H_i \cdot L_i} \quad [3.3-4]$$

where,

ρ_i – density of the gas space in the i^{th} cavity at temperature $T_{gap,i}$

$c_{p,i}$ – specific heat capacity of the gas space in the i^{th} cavity at temperature $T_{gap,i}$

$T_{gap,i,in}$ – temperature at the inlet of the glazing cavity that depends on where the air comes from

$T_{gap,i,out}$ – temperature at the outlet of the glazing cavity (see equation 3.3-15)

H_i – height of the i^{th} glazing cavity (see **Figure 3-8**)

L_i – depth of the i^{th} glazing cavity (see **Figure 3-8**)

$\varphi_{v,i}$ – air flow rate in the i^{th} cavity, given as:

$$\varphi_{v,i} = v_i \cdot s_i \cdot L_i \quad [3.3-5]$$

where,

s_i – width of the i^{th} glazing cavity (see **Figure 3-8**)

Using the equation [3.3–1], the total flux (including radiative part) leaving the front facing surface of the i^{th} layer can be determined as:

$$q_i = q_{cf,i} + J_{f,i} - J_{b,i-1} = h_{cv,i}(T_{f,i} - T_{gap,i}) + J_{f,i} - J_{b,i-1} \quad [3.3-6]$$

Similarly, the heat flux leaving the layer $i+1$ is:

$$q_{i+1} = q_{cf,i+1} + J_{f,i+1} - J_{b,i} = h_{cv,i+1}(T_{f,i+1} - T_{gap,i+1}) + J_{f,i+1} - J_{b,i} \quad [3.3-7]$$

Finally, as a result of the mentioned modifications, made due to effects of ventilation on heat exchange in the glazing cavity, the basic energy balance equations for shading devices can be defined as the following system:

$$q_i = S_i + q_{i+1} + q_{v,i+1} \quad [3.3-8]$$

$$J_{f,i} = \varepsilon_{f,i} \sigma T_{f,i}^4 + \tau_i J_{f,i+1} + \rho_{f,i} J_{b,i-1} \quad [3.3-9]$$

$$J_{b,i} = \varepsilon_{b,i} \sigma T_{b,i}^4 + \tau_i J_{b,i-1} + \rho_{b,i} J_{f,i+1} \quad [3.3-10]$$

$$T_{b,i} - T_{f,i} = \frac{t_{sd,i}}{2k_{sd,i}} [2q_{i+1} + 2q_{v,i+1} + S_i] \quad [3.3-11]$$

3.3.1.1. CALCULATION OF GLAZING CAVITY TEMPERATURE

The temperature in the glazing cavity can be determined by applying assumption about known value of mean velocity in the cavity to a model, which considers dependence of the temperature on the glazing cavity height (see **Figure 3-9**).

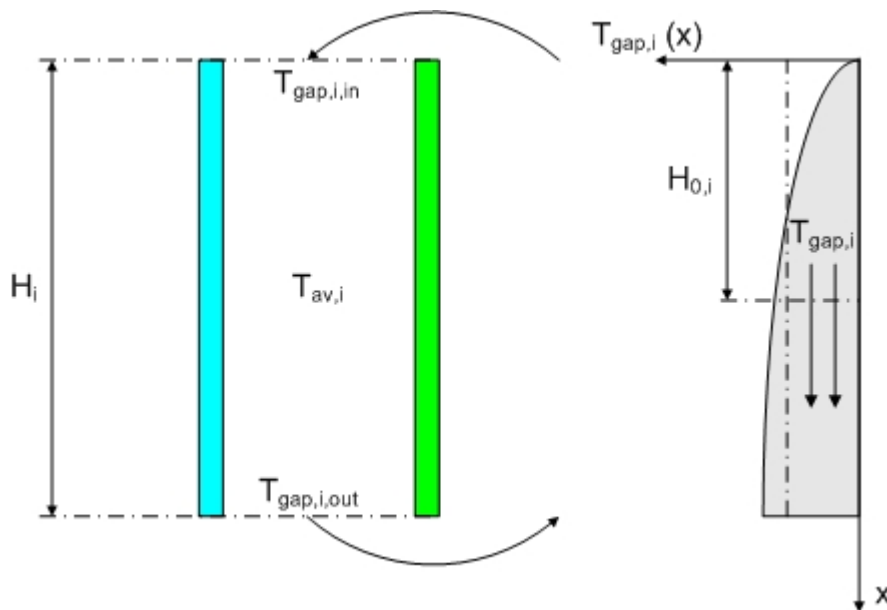


Figure 3-9: Temperature Profile in Ventilated Glazing Cavity

Temperature profile in the ventilated glazing cavity can be described as:

$$T_{gap,i}(x) = T_{av,i} - (T_{av,i} - T_{gap,i,in})e^{-x/H_{0,i}} \quad [3.3-12]$$

where,

x – distance from the inlet

$T_{gap,i}(x)$ – temperature in the glazing cavity at distance x

$T_{av,i}$ – average temperature of the layer surfaces bounding glazing cavity, given as:

$$T_{av,i} = \frac{T_{b,i-1} + T_{f,i}}{2} \quad [3.3-13]$$

$T_{gap,i,in}$ – air flow temperature at the inlet of the ventilated gap

$H_{0,i}$ – characteristic height (temperature penetration length) defined by:

$$H_{0,i} = \frac{\rho_i \cdot c_p \cdot s_i \cdot v_i}{2h_{cv,i}} \quad [3.3-14]$$

where,

ρ_i – density of the air at temperature $T_{gap,i}$

$c_{p,i}$ – specific heat capacity of the air at temperature $T_{gap,i}$

s_i – width of the i^{th} glazing cavity (see **Figure 3-8**)

v_i – mean air velocity in the glazing cavity (see section **3.3.1.2**)

$h_{cv,i}$ – convection heat transfer coefficient for ventilated cavities, presented in equation [3.3-3]

The following relation, which determines temperature of air flow leaving the ventilated glazing cavity, is obtained from the equation [3.3-12] by setting the glazing cavity height, H_i , for the distance from the inlet:

$$T_{gap,i,out} = T_{av,i} - (T_{av,i} - T_{gap,i,in})e^{-H_i/H_{0,i}} \quad [3.3-15]$$

The previous equation clearly shows that temperature of air flow leaving the ventilated cavity (i.e. $T_{gap,i,out}$) depends on temperature at the inlet of the glazing cavity (i.e. $T_{gap,i,in}$). In case of connected glazing cavities (as given in **Figure 3-11**), it means that the temperature at the inlet of i^{th} cavity (i.e. $T_{gap,i,out}$) is a function of the temperature at the outlet of $i+1^{\text{th}}$ cavity, and vice versa. For the solution of this problem, the following model, shown in **Figure 3-10**, should be introduced.

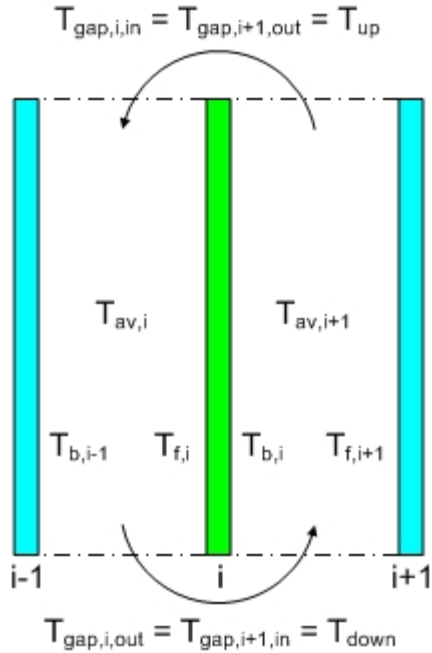


Figure 3-10: Model For Solving Inlet and Outlet Temperatures in Ventilated Cavities

Applying the notation from the **Figure 3-10** in equation [3.3–15] that is set for both ventilated cavities, gives the following relations:

$$T_{gap,i,out} = T_{down} = T_{av,i} - (T_{av,i} - T_{up})e^{-H_i/H_{0,i}} \quad [3.3-16]$$

$$T_{gap,i+1,out} = T_{up} = T_{av,i+1} - (T_{av,i+1} - T_{down})e^{-H_{i+1}/H_{0,i+1}} \quad [3.3-17]$$

After some re-arrangements, the following relations could be derived from the equations [3.3–16] and [3.3–17]:

$$T_{down} - \beta_i T_{up} = \alpha_i T_{av,i} \quad [3.3-18]$$

$$T_{up} - \beta_{i+1} T_{down} = \alpha_{i+1} T_{av,i+1} \quad [3.3-19]$$

where the terms α_i , α_{i+1} , β_i and β_{i+1} are:

$$\alpha_i = 1 - e^{-\frac{H_i}{H_{0,i}}} \quad [3.3-20]$$

$$\alpha_{i+1} = 1 - e^{-\frac{H_{i+1}}{H_{0,i+1}}} \quad [3.3-21]$$

$$\beta_i = e^{-\frac{H_i}{H_{0,i}}} \quad [3.3-22]$$

$$\beta_{i+1} = e^{-\frac{H_{i+1}}{H_{0,i+1}}} \quad [3.3-23]$$

By solving the equations [3.3–18] and [3.3–19], temperatures at the inlet and outlet of ventilated glazing gaps can be found as:

$$T_{down} = T_{gap,i,out} = T_{gap,i+1,in} = \frac{\alpha_i T_{av,i} + \beta_i \alpha_{i+1} T_{av,i+1}}{1 - \beta_i \beta_{i+1}} \quad [3.3-24]$$

$$T_{up} = T_{gap,i,in} = T_{gap,i+1,out} = \alpha_{i+1} T_{av,i+1} + \beta_{i+1} \frac{\alpha_i T_{av,i} + \beta_i \alpha_{i+1} T_{av,i+1}}{1 - \beta_i \beta_{i+1}} \quad [3.3-25]$$

The above procedure is done in case of higher temperature in the cavity ***i+1*** (i.e. $T_{gap,i+1}$) than in the connected space ***i*** (i.e. $T_{gap,i}$).

Otherwise (i.e. if $T_{gap,i} > T_{gap,i+1}$), inlet and outlet temperatures for ventilated glazing cavities are:

$$T_{up} = T_{gap,i,out} = T_{gap,i+1,in} = \frac{\alpha_i T_{av,i} + \beta_i \alpha_{i+1} T_{av,i+1}}{1 - \beta_i \beta_{i+1}} \quad [3.3-26]$$

$$T_{down} = T_{gap,i,in} = T_{gap,i+1,out} = T_{av,i+1} \alpha_{i+1} + \beta_{i+1} \frac{\alpha_i T_{av,i} + \beta_i \alpha_{i+1} T_{av,i+1}}{1 - \beta_i \beta_{i+1}} \quad [3.3-27]$$

Finally, the equivalent average temperatures of the ventilated glazing cavities – $T_{gap,i}$ and $T_{gap,i+1}$ can be determined from the equation [3.3–12] by integration over the glazing cavity heights – H_i and H_{i+1} , respectively.

$$T_{gap,i} = \frac{1}{H_i} \int_0^H T_{gap,i}(x) dx = T_{av,i} - \frac{H_{0,i}}{H_i} (T_{gap,i,out} - T_{gap,i,in}) \quad [3.3-28]$$

$$T_{gap,i+1} = \frac{1}{H_{i+1}} \int_0^H T_{gap,i+1}(x) dx = T_{av,i+1} - \frac{H_{0,i+1}}{H_{i+1}} (T_{gap,i+1,out} - T_{gap,i+1,in}) \quad [3.3-29]$$

3.3.1.2. CALCULATION OF AIR VELOCITY IN GLAZING CAVITY

Procedure for calculation of air flow velocity depends on nature of ventilation in the glazing cavity. Thus, different calculation methods are used in cases of thermally driven ventilation and forced ventilation.

3.3.1.2.1. Thermally Driven Ventilation

In this case air flow is caused by the stack or buoyancy effect, and the velocity can be found as a function of the driving pressure difference between connected spaces and the resistance to the air flow of the openings and the gas spaces itself.

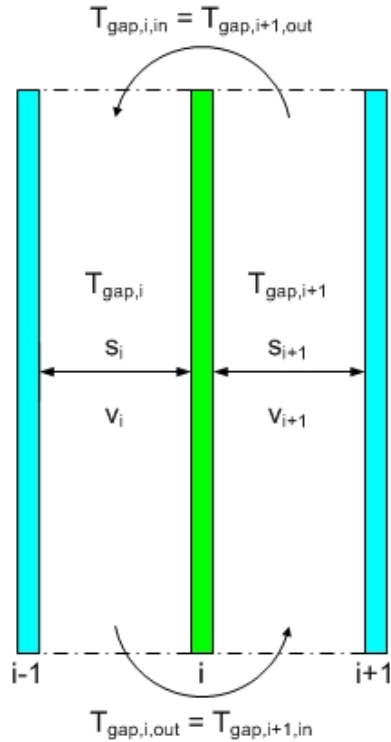


Figure 3-11: Schematic Presentation of the Thermally Driven Ventilation

The temperature difference between the glazing cavity, i , and the connected space, which can be either the outdoor or indoor environment, or another glazing cavity (as displayed in **Figure 3-11**), produces the driving pressure difference, $\Delta p_{T,i-i+1}$, that can be expressed as:

$$\Delta p_{T,i-i+1} = \rho_0 \cdot T_0 \cdot g \cdot H_i \cos \gamma_i \cdot \frac{|T_{gap,i} - T_{gap,i+1}|}{T_{gap,i} \cdot T_{gap,i+1}} \quad [3.3-30]$$

where,

ρ_0 – density of the air at reference temperature T_0

T_0 – reference temperature (283 K)

g – acceleration due to gravity

H_i – glazing cavity height

γ_i – glazing system inclination (i.e. tilt angle) from vertical

$T_{gap,i}$ – mean temperature of the glazing cavity, given by equation [3.3–28]

$T_{gap,i+1}$ – mean temperature of the connected space (outdoor or indoor environment, or another glazing cavity)

The driving pressure difference $\Delta p_{T,i-i+1}$ shall be equal to the total pressure loss, which includes:

- Bernoulli pressure losses in spaces ***i*** and ***i+1***, defined as:

$$\Delta p_{B,i} = 0.5 \rho_i v_i^2 \quad [3.3-31]$$

$$\Delta p_{B,i+1} = 0.5 \rho_{i+1} v_i^2 \left(\frac{s_i}{s_{i+1}} \right)^2 \quad [3.3-32]$$

- Hagen – Poiseuille pressure losses in spaces ***i*** and ***i+1***, defined as:

$$\Delta p_{HP,i} = 12 \mu_i \frac{H_i}{s_i^2} v_i \quad [3.3-33]$$

$$\Delta p_{HP,i+1} = 12 \mu_{i+1} \frac{H_{i+1} \cdot s_i}{s_{i+1}^3} v_i \quad [3.3-34]$$

- pressure losses at the inlet and outlet of spaces ***i*** and ***i+1***, defined as:

$$\Delta p_{Z,i} = 0.5 \rho_i v_i^2 (Z_{in,i} + Z_{out,i}) \quad [3.3-35]$$

$$\Delta p_{Z,i+1} = 0.5 \rho_{i+1} v_i^2 (Z_{in,i+1} + Z_{out,i+1}) \left(\frac{s_i}{s_{i+1}} \right)^2 \quad [3.3-36]$$

where,

ρ_i, ρ_{i+1} – density of the air at temperatures $T_{gap,i}$ and $T_{gap,i+1}$, respectively

v_i – mean air velocity in the i^{th} glazing cavity that is being sought

H_i, H_{i+1} – height of i^{th} and $i+1^{th}$ glazing cavity, respectively

s_i, s_{i+1} – width of i^{th} and $i+1^{th}$ glazing cavity, respectively (see **Figure 3-8**)

μ_i, μ_{i+1} – dynamic viscosity of the air at temperatures $T_{gap,i}$ and $T_{gap,i+1}$, respectively

Z_i, Z_{i+1} – pressure loss factors of the i^{th} and $i+1^{th}$ glazing cavity, found as per equations [3.3-37] – [3.3-40].

Note that all pressure losses for the space ***i+1*** are given in terms of air velocity in the space ***i*** (i.e. v_i) by setting velocity in the space ***i+1*** as: $v_{i+1} = v_i \cdot s_i / s_{i+1}$.

The pressure loss factors, Z_i and Z_{i+1} , can be found using the ratio of the equivalent opening areas ($A_{eq,i}$ and $A_{eq,i+1}$) to the cross sections of corresponding gas spaces ($A_{s,i}$ and $A_{s,i+1}$).

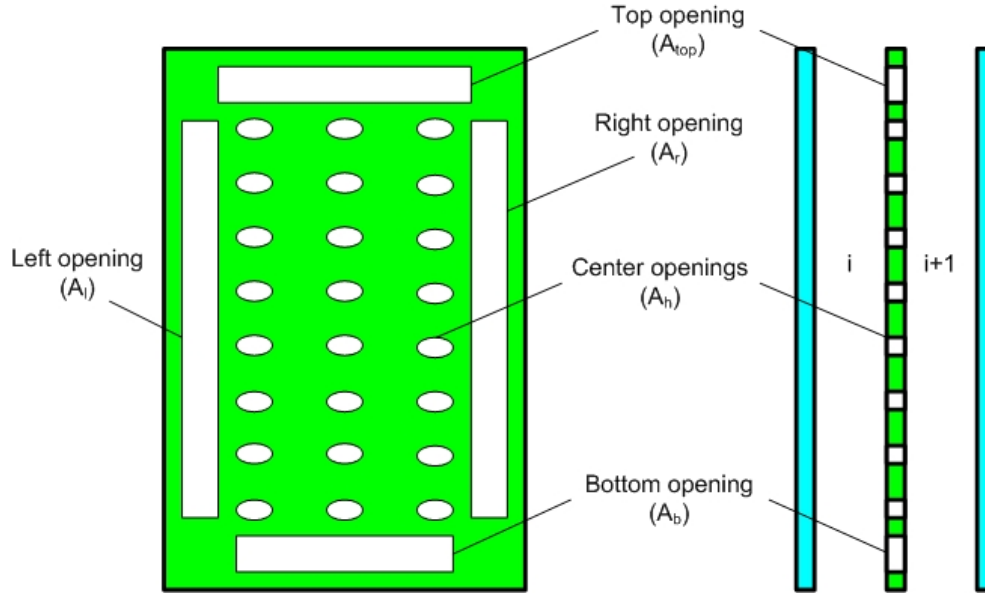


Figure 3-12: Openings in the Ventilated Glazing Cavity

$$Z_{in,i} = \left(\frac{A_{s,i}}{0.6A_{eq,in,i}} - 1 \right)^2 \quad [3.3-37]$$

$$Z_{in,i+1} = \left(\frac{A_{s,i+1}}{0.6A_{eq,in,i+1}} - 1 \right)^2 \quad [3.3-38]$$

$$Z_{out,i} = \left(\frac{A_{s,i}}{0.6A_{eq,out,i}} - 1 \right)^2 \quad [3.3-39]$$

$$Z_{out,i+1} = \left(\frac{A_{s,i+1}}{0.6A_{eq,out,i+1}} - 1 \right)^2 \quad [3.3-40]$$

where,

$A_{s,i}$, $A_{s,i+1}$ – cross section of the i^{th} and $i+1^{th}$ glazing cavity, respectively ($A_{s,i} = s_i * L_i$; $A_{s,i+1} = s_{i+1} * L_{i+1}$)

s_i , s_{i+1} – width of i^{th} and $i+1^{th}$ glazing cavity, respectively (see **Figure 3-8**)

L_i , L_{i+1} – depth of the i^{th} and $i+1^{th}$ glazing cavity, respectively (see **Figure 3-8**)

Since the spaces ***i*** and ***i+1*** are connected, equivalent inlet opening area of the ***i***th glazing cavity is, in fact, equivalent outlet opening area of the ***i+1***th glazing cavity. Consequently, equivalent outlet opening area of the ***i***th glazing cavity is equal to the equivalent inlet opening area of ***i+1***th glazing cavity.

Calculation of equivalent inlet and outlet opening areas depends on temperature difference between adjacent glazing cavities (i.e. spaces ***i*** and ***i+1***). If the temperature

of the cavity i (i.e. $T_{gap,i}$) is higher than the temperature of the connected space $i+1$ (i.e. $T_{gap,i+1}$), equivalent opening areas are:

$$A_{eq,in,i} = A_{eq,out,i+1} = A_{bot} + 0.5 \cdot \frac{A_{top}}{A_{bot} + A_{top}} (A_l + A_r + A_h) \quad [3.3-41]$$

$$A_{eq,out,i} = A_{eq,in,i+1} = A_{top} + 0.5 \cdot \frac{A_{bot}}{A_{bot} + A_{top}} (A_l + A_r + A_h) \quad [3.3-42]$$

Otherwise,

$$A_{eq,in,i} = A_{eq,out,i+1} = A_{top} + 0.5 \cdot \frac{A_{bot}}{A_{bot} + A_{top}} (A_l + A_r + A_h) \quad [3.3-43]$$

$$A_{eq,out,i} = A_{eq,in,i+1} = A_{bot} + 0.5 \cdot \frac{A_{top}}{A_{bot} + A_{top}} (A_l + A_r + A_h) \quad [3.3-44]$$

Finally, air velocity in the space i (i.e. v_i) can be found from the following equation, which defines driving pressure difference (i.e. $\Delta p_{T,i-i+1}$) in terms of pressure losses.

$$\Delta p_{T,i-i+1} = \Delta p_{B,i} + \Delta p_{HP,i} + \Delta p_{Z,i} + \Delta p_{B,i+1} + \Delta p_{HP,i+1} + \Delta p_{Z,i+1} \quad [3.3-45]$$

After applying relation [3.3-30] for the driving pressure difference, as well as relations [3.3-31] – [3.3-36] for pressure losses, and some re-arrangements, the equation [3.3-45] becomes:

$$A_1 \cdot v_i^2 + A_2 \cdot v_i - A = 0 \quad [3.3-46]$$

where the terms A , A_1 and A_2 are:

$$A = \rho_0 \cdot T_0 \cdot g \cdot H_i \cos \gamma_i \cdot \frac{|T_{gap,i} - T_{gap,i+1}|}{T_{gap,i} \cdot T_{gap,i+1}} \quad [3.3-47]$$

$$A_1 = 0.5 \left[\rho_i (1 + z_{in,i} + z_{out,i}) + \rho_{i+1} \left(\frac{s_i}{s_{i+1}} \right)^2 (1 + z_{in,i+1} + z_{out,i+1}) \right] \quad [3.3-48]$$

$$A_2 = 12 \left(\mu_i \frac{H_i}{s_i^2} + \mu_{i+1} \frac{H_{i+1} \cdot s_i}{s_{i+1}^3} \right) \quad [3.3-49]$$

The air velocity v_i , as solution of the quadratic equation [3.3-46], is given by the following relation:

$$v_i = \frac{\sqrt{A_2^2 + |4 \cdot A \cdot A_1|} - A_2}{2A_1} \quad [3.3-50]$$

If the space $i+1$ is exterior or interior environment, velocity v_{i+1} is set to zero, and the pressure losses $\Delta p_{B,i+1}$, $\Delta p_{HP,i+1}$ and $\Delta p_{Z,i+1}$ become zero, as well. In that case, the quadratic equation terms A_1 and A_2 will be:

$$A_1 = 0.5\rho_i(1 + z_{in,i} + z_{out,i}) \quad [3.3-51]$$

$$A_2 = 12\mu_i \frac{H_i}{s_i^2} \quad [3.3-52]$$

3.3.1.2.2. Forced Ventilation

In the case of forced ventilation, air flow velocity (i.e. v_i) can be determined from the known value of air flow rate, using the following relation:

$$v_i = \frac{\varphi_{v,i}}{s_i \cdot L_i} \quad [3.3-53]$$

where,

s_i – width of the glazing cavity (see **Figure 3-8**)

L_i – length of the glazing cavity (see **Figure 3-8**)

$\varphi_{v,i}$ – air flow rate in the glazing cavity (for the whole area), not normalized per m^2

3.3.2. System of Equations for Glazing Systems Incorporating Shading Devices

The non-linear system of energy balance equations for shading devices, given in equations [3.3–8] – [3.3–11], can become linear if solved in terms of black emissive power instead of temperature. In this case, differences from glazing systems with monolithic layers are related to convective heat transfer through ventilated cavities, so the heat transfer coefficient based on emissive power for ventilated cavities must be introduced, instead of the previous one (i.e. for non-ventilated cavities), given in equation [3.1–7]:

$$\hat{h}_{v,i} = h_{cv,i} \frac{T_{f,i} - T_{gap,i}}{E_{bf,i} - E_{bgap,i}} \quad [3.3-54]$$

Applying the methodology, described at the beginning of section Error! Reference source not found., to monolithic layers, in conjunction with relations [3.3–8] – [3.3–11] and [3.3–54] for shading layers, gives the following system of linear equations for glazing systems incorporating shading devices:

$$J_{f,1} + \hat{h}^{out} E_{bf,1} + \hat{h}_2 E_{bb,1} + J_{b,1} - J_{f,2} - \hat{h}_2 E_{bf,2} = S_1 + G_{out} + \hat{h}^{out} G_{out} \quad [3.3-55]$$

$$-J_{f,1} + \varepsilon_{f,1} E_{bf,1} + \tau_1 J_{f,2} = -\rho_{f,1} G_{out} \quad [3.3-56]$$

$$\varepsilon_{b,1} E_{bb,1} - J_{b,1} + \rho_{b,1} J_{f,2} = -\tau_1 G_{out} \quad [3.3-57]$$

$$\hat{h}_1^{gl} E_{bf,1} + (\hat{h}_1^{gl} + \hat{h}_2) E_{bb,1} + J_{b,1} - J_{f,2} - \hat{h}_2 E_{bf,2} = 0.5 S_1 \quad [3.3-58]$$

...

$$-J_{b,i-1} + J_{f,i} + \hat{h}_{v,i} E_{bf,i} + J_{b,i} - J_{f,i+1} - \hat{h}_{v,i+1} E_{bf,i+1} = S_i + \hat{h}_{v,i} E_{bgap,i} - \hat{h}_{v,i+1} E_{bgap,i+1} + q_{v,i+1} \quad [3.3-59]$$

$$\rho_{f,i} J_{b,i-1} - J_{f,i} + \varepsilon_{f,i} E_{bf,i} + \tau_i J_{f,i+1} = 0 \quad [3.3-60]$$

$$\tau_i J_{b,i-1} + \varepsilon_{b,i} E_{bb,i} - J_{b,i} + \rho_{b,i} J_{f,i+1} = 0 \quad [3.3-61]$$

$$-\hat{h}_i^{gl} E_{bf,i} + \hat{h}_i^{gl} E_{bb,i} + J_{b,i} - J_{f,i+1} - \hat{h}_{v,i+1} E_{bf,i+1} = 0.5 S_i + q_{v,i+1} - \hat{h}_{v,i+1} E_{bgap,i+1} \quad [3.3-62]$$

...

$$-\hat{h}_n E_{bb,n-1} - J_{b,n-1} + J_{f,n} + \hat{h}_n E_{bf,n} + \hat{h}^{in} E_{bb,n} + J_{b,n} = S_n + G_{in} + \hat{h}^{in} G_{in} \quad [3.3-63]$$

$$\rho_{f,n} J_{b,n-1} - J_{f,n} + \varepsilon_{f,n} E_{bf,n} = -\tau_n G_{in} \quad [3.3-64]$$

$$\tau_n J_{b,n-1} + \varepsilon_{b,n} E_{bb,n} - J_{b,n} = -\rho_{b,n} G_{in} \quad [3.3-65]$$

$$\hat{h}_n^{gl} E_{bf,n} + (\hat{h}_n^{gl} + \hat{h}^{in}) E_{bb,n} + J_{b,n} = 0.5 S_n + G_{in} + \hat{h}^{in} G_{in} \quad [3.3-66]$$

The system of equations [3.3-55] – [3.3-66] can also be transformed into a matrix form $[A] [X] = [B]$ for the whole glazing system.

$$[A] = \begin{bmatrix} 1 & \hat{h}^{out} & \hat{h}_2 & 1 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ -1 & \varepsilon_{f,1} & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_{b,1} & -1 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & -\hat{h}_1^{gl} & \hat{h}_1^{gl} + \hat{h}_2 & 1 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 1 & \hat{h}_{v,i} & 0 & 1 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & -1 & \varepsilon_{f,i} & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & \varepsilon_{b,i} & -1 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & -\hat{h}_i^{gl} & \hat{h}_i^{gl} & 1 & \dots & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 1 & \hat{h}_n & \hat{h}^{in} & 1 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & -1 & \varepsilon_{f,n} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 & \varepsilon_{b,n} & -1 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & -\hat{h}_n^{gl} & \hat{h}_n^{gl} + \hat{h}^{in} & 1 \end{bmatrix}$$

$$[X] = \begin{bmatrix} J_{f,1} \\ E_{bf,1} \\ E_{bb,1} \\ J_{b,1} \\ \dots \\ J_{f,i} \\ E_{bf,i} \\ E_{bb,i} \\ J_{b,i} \\ \dots \\ J_{f,n} \\ E_{bf,n} \\ E_{bb,n} \\ J_{b,n} \end{bmatrix} \quad [B] = \begin{bmatrix} S_1 + G_{out} + \hat{h}^{out} G_{out} \\ -\rho_{f,1} G_{out} \\ -\tau_1 G_{out} \\ 0.5S_1 \\ \dots \\ S_i + \hat{h}_{v,i} E_{bgap,i} - \hat{h}_{v,i+1} E_{bgap,i+1} + q_{v,i+1} \\ 0 \\ 0 \\ 0.5S_i + q_{v,i+1} - \hat{h}_{v,i+1} E_{bgap,i+1} \\ \dots \\ S_n + G_{in} + \hat{h}^{in} G_{in} \\ -\tau_n G_{in} \\ -\rho_{b,n} G_{in} \\ 0.5S_n + G_{in} + \hat{h}^{in} G_{in} \end{bmatrix}$$

The matrix equation $[A] [X] = [B]$ is solved and the glazing layer temperatures determined, using the same iterative procedure as for glazing systems with monolithic layers (see sections 3.1.2.4 and 3.1.2.5).

3.3.3. Actual Cavity Width Convection Model:

In this model, convection around the SD is treated as if shading device is just another impermeable layer, and the width of this layer and the corresponding gap space width between this layer and the glass is assumed to be equal to the actual width. This means that if the SD is venetian blind and if the venetian blind is open, the width of the venetian blind layer is assumed to be the width from the tip to tip of venetian blind slats. For example, if 16 mm wide venetian blinds are installed in 20 mm glazing cavity, and if venetian blind is at 0 degrees angle, which means that slats are horizontal (fully opened), the venetian blind layer width will be 16 mm, while each gap around the venetian blind will be 2 mm wide. If on the other hand that same blind is at the 45 degree, the width of the venetian blind layer will be $16 \cdot \cos(45) = 11.31$ mm, while each gap around the blind will be 4.34 mm. For the fully closed blind, the thickness of the venetian blind layer will be the thickness of the blind material.

Treatment of SD on indoor and outdoor side is similar, where actual cavity width is calculated by subtracting half the width of the shading device (tip to tip) from the distance of the center of SD to the surface of the glass.

Conductivity of such layer is ignored, meaning that thermal resistance of the layer itself is negligible. This model has been compared with the limited set of measurements from published technical papers and it gave reasonable agreement.

3.3.4. Scalar Convection

Scalar convection is an alternative methodology for calculating convective heat transfer in the presence of shading devices. In this model, the convection in gaps around the SD is treated as an intermediate case between two extreme cases, set with the use of convection scalar, with range from 0 to 1. The two extreme cases are:

1. Existence of SD is ignored for the convection heat transfer and convection equations for unobstructed glazing cavity are solved in each iteration.
2. SD is assumed to be fully closed and is treated as impermeable layer with the negligible thickness. Two gaps are formed as a result and convection heat transfer is solved as if solid layer is placed between two glass layers (making effectively triple glazing out of double glazing with SD in between).

When convective scalar is set to 0, case 1 is solved. When convective scalar is set to 1, the case 2 is solved. When convective scalar is set to the value in-between 0 and 1 (e.g., 0.5), then linear interpolation between the two cases is performed at each iteration.

Similar procedure is employed for indoor and outdoor venetian blind, except that blind in those cases form one additional cavity on indoor or outdoor, respectively.

3.3.4.1. "No SD" Case

Case "No SD" treats glazing system without SD layer in terms of convection. As a first step, SD layer is removed from the glazing system and calculation is performed for two glass layers (**Error! Reference source not found.**) where gap thickness S_{gg} equals a distance between first and third layer (sum of first and second gap thickness and SD thickness).

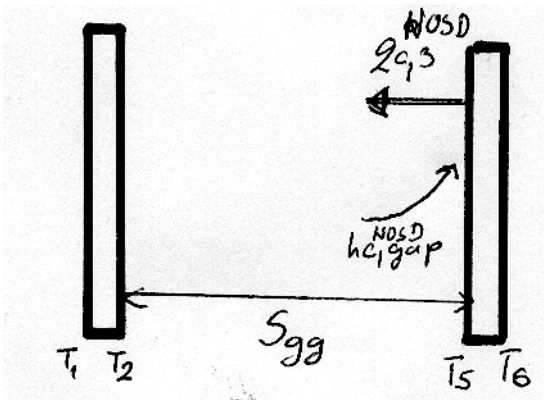


Figure 3-13: Glazing system with two glass glazing pane

Convective heat transfer coefficient for this case depends on glazing layer temperatures and gap thickness:

$$h_{c,gap}^{NOSD} = f(T_2, T_5, S_{gg}) = N_{u,i} \cdot \frac{k_g}{S_{gg}} \quad [3.3-67]$$

where:

- S_{gg} is thickness of glazing cavity which is equal to the sum of thickness of first, second gap and SD thickness.
- k_g is thermal conductivity of the fill gas in the cavity. Convective heat flux through the glazing cavity is determined as:

$$q_{c,3}^{NOSD} = h_{c,gap}^{NOSD} \cdot (T_5 - T_2) \quad [3.3-68]$$

In the original configuration, with SD present, convective parts of heat fluxes in two gaps can be expressed as (**Figure 3-14**):

$$q_{c,2}^o = h_{c,gap1}^o \cdot (T_3 - T_2) \quad [3.3-69]$$

$$q_{c,3}^o = h_{c,gap2}^o \cdot (T_5 - T_4) \quad [3.3-70]$$

In order to form a glazing system in which existence of SD layer does not affect convective heat transfer (**Figure 3-14**), the resulting convective parts of heat fluxes in two gaps, expressed as in eq. (**Error! Reference source not found.**) and (**Error! Reference source not found.**), must be equal to heat flux from the “no SD” configuration:

$$q_{c,2}^o = q_{c,3}^{NOSD} \quad [3.3-71]$$

$$q_{c,3}^o = q_{c,3}^{NOSD} \quad [3.3-72]$$

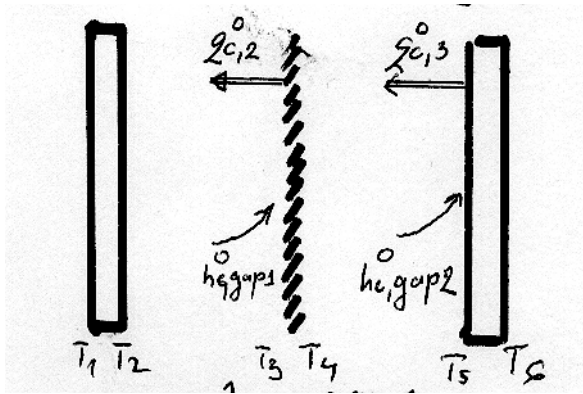


Figure 3-14: Glazing system without SD influence on convection

Therefore, convective heat transfer coefficients for a “No SD” case become:

$$h_{c,gap1}^o = h_{c,gap2}^o = h_{c,gap}^{NOSD} \quad [3.3-73]$$

3.3.4.2. “Closed SD” Case

“Closed SD” case (**Figure 3-15**) treats SD layer as closed, which means it will be treated as glass layer with SD thickness and conductivity of SD material. Glazing cavity convective heat transfer coefficients $h_{c,gap1}^1$ and $h_{c,gap2}^1$ will be calculated in a “standard”

way, as in a case of two glass layers with a gap between them – as explained in Section 3.3.4.1 – equation (I).

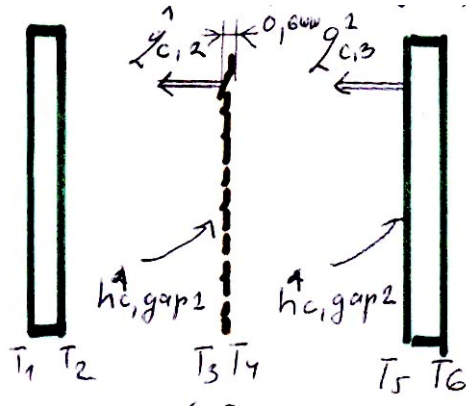


Figure 3-15: Glazing system with closed SD which is treated as glass layer

Combination of the extreme cases “NO SD” and “Closed SD” which were presented above will be obtained by introducing new scalar parameter – Alpha:

$$h_{c,gap1}^{\alpha} = \alpha \cdot (h_{c,gap1}^1 - h_{c,gap1}^0) + h_{c,gap1}^0 \quad [3.3-74]$$

$$h_{c,gap2}^{\alpha} = \alpha \cdot (h_{c,gap2}^1 - h_{c,gap2}^0) + h_{c,gap2}^0 \quad [3.3-75]$$

These convective heat transfer coefficients will be used in “standard” energy balance equations instead of convection heat transfer coefficients for ventilated cavities $h_{cv,1}$ and $h_{cv,2}$, so equation [3.3-54] will become:

$$\hat{h}_{gap1}^{\alpha} = h_{c,gap1}^{\alpha} \cdot \frac{T_3 - T_2}{E_{bf,2} - E_{bb,1}} \quad [3.3-76]$$

$$\hat{h}_{gap2}^{\alpha} = h_{c,gap2}^{\alpha} \cdot \frac{T_5 - T_4}{E_{bf,3} - E_{bb,2}} \quad [3.3-77]$$

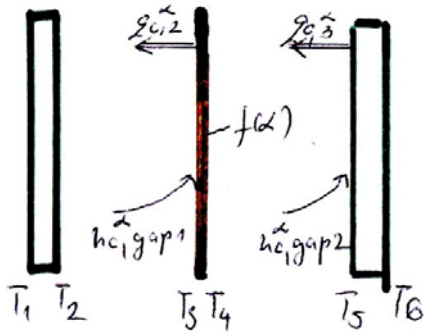


Figure 16. Glazing system with new convection coefficients

When factor Alpha is set to zero new model will calculate convection as if there was no SD layer. If Alpha is set to one, convective coefficient will correspond to second extreme case – “Closed SD”.

3.3.5. Calculation of U – factor and SHGC

The U – factor of glazing systems with shading devices is calculated using essentially the same method as for other glazing system types (i.e. with monolithic or laminated layers), since it is, again, defined as the reciprocal of the total glazing system thermal resistance – R_{tot} . As for other glazing system types, the R_{tot} is, also, determined as a sum of the thermal resistance on the outdoor side of the glazing system, thermal resistances of the glazing layers and glazing cavities and the thermal resistance on the indoor side of the glazing system.

The thermal resistance on the glazing system outdoor side – R_{out} , thermal resistances of glazing layers – $R_{gl,i}$ and thermal resistance on the glazing system indoor side – R_{in} , are calculated using the same relations as for glazing system with monolithic layers (i.e. 3.1–95, 3.1–96 and 3.1–98, respectively). At the same time, thermal resistance – $R_{gap,i}$, of ventilated glazing cavities is calculated differently from the thermal resistance of non-ventilated cavities, given in equation [3.1–97]. In case of ventilated cavities, heat transfer in the cavities is divided into two parts due to ventilation and, therefore, thermal resistance of ventilated cavities should be expressed in the same way.

$$R_{gap,i} = R_{gap,f,i} + R_{gap,b,i-1} \quad [3.3-78]$$

where,

$R_{gap,f,i}$ – resistance to heat transfer between the glazing layer and ventilated cavity, given as:

$$R_{gap,f,i} = \frac{T_{f,i} - T_{gap,i}}{h_{cv,i}(T_{f,i} - T_{gap,i}) + J_{f,i} - J_{b,i-1}} \quad [3.3-79]$$

$R_{gap,b,i-1}$ – resistance to heat flow between the ventilated cavity and another glazing layer, given as:

$$R_{gap,b,i-1} = \frac{T_{gap,i} - T_{b,i-1}}{h_{cv,i}(T_{gap,i} - T_{b,i-1}) + J_{f,i} - J_{b,i-1}} \quad [3.3-80]$$

The calculation of solar heat gain coefficient (SHGC) is performed in the same manner as for glazing systems with monolithic layers using the equation [3.1–99].

4. ISO/EN 10077-1 Algorithms

ISO/EN 10077-1 algorithms are substantially simplified over the ISO 15099 algorithms. These algorithms are used in new product standards in EU and associated countries. This standard relies on additional standards ISO 10292/EN673 (specular glazing layers) and EN 13663 (shading devices).

4.1. Specular Glazing Layers (EN673/ISO10292)

4.1.1. Definition of Outdoor and Indoor Heat Transfer Coefficients

The value of **outdoor heat transfer coefficient** (i.e. h_e) depends on the wind speed near the glazing system and other climate factors, as well as on the outdoor glazing surface emissivity. The EN 673 standard does not consider coated outdoor glazing surfaces, which emissivities are lower than 0.840, and the value of h_e is standardized to:

$$h_e = 23 \frac{W}{m^2 K} \quad [4.1-1]$$

At the same time, the **indoor heat transfer coefficient** (i.e. h_i) is defined by following equation:

$$h_i = h_{r,i} + h_{c,i} \quad [4.1-2]$$

where,

$h_{r,i}$ – radiation conductance

$h_{c,i}$ – convection conductance

The radiation conductance of uncoated indoor glazing surfaces is set to 4.4 W/m²K, while for the coated ones it is given as:

$$h_{r,i} = \frac{4.4\varepsilon_i}{0.840} \quad [4.1-3]$$

where,

ε_i – emissivity of coated indoor glazing surface

Finally, free convection is assumed to be on the indoor side of the glazing system, so the value of convection conductance (i.e. $h_{c,i}$) is:

$$h_{c,i} = 3.6 \frac{W}{m^2 K} \quad [4.1-4]$$

4.2. Calculation of Glazing Cavity Thermal Conductance

The thermal conductance of glazing cavity can be determined by summing the convective and radiative components. The convection component is defined in terms of the gas conductance (i.e. $h_{gs,i}$), while the radiation conductance of glazing cavity (i.e. $h_{rs,i}$) determines the radiative component.

$$h_{s,i} = h_{rs,i} + h_{gs,i} \quad [4.2-1]$$

4.2.1. Gas Conductance

The gas conductance of the glazing cavity can be determined according to the following relation:

$$h_{gs,i} = N_{u,i} \frac{\lambda_i}{s_i} \quad [4.2-2]$$

where,

$N_{u,i}$ – Nusselt number

λ_i – thermal conductivity of the fill gas in the glazing cavity

s_i – width of the glazing cavity

Nusselt number, which is a function of Grashof number, Prandtl number and glazing system inclination (i.e. tilt angle), is calculated as:

$$N_{u,i} = A(G_r \cdot P_r)^n \quad [4.2-3]$$

where,

A – constant

G_r – Grashof number

P_r – Prandtl number

n – exponent

If the calculated value of the Nusselt number is less than 1, it is set to the bounding value of 1.

The Nusselt number dependence on the glazing system tilt angle is given through the following relations for the constant A and exponent n :

Glazing system inclined at 0° ($\theta = 0^\circ$)

$$A = 0.16 ; n = 0.28 \quad [4.2-4]$$

Glazing system inclined at 45° ($\theta = 45^\circ$)

$$A = 0.10 ; n = 0.31 \quad [4.2-5]$$

Glazing system inclined at 90° ($\theta = 90^\circ$)

$$A = 0.035 ; n = 0.38 \quad [4.2-6]$$

The Grashof and Prandtl numbers can be expressed as:

$$G_r = \frac{9.81 \cdot s_i^3 \cdot \Delta T_i \cdot \rho_i^2}{T_{m,i} \cdot \mu_i^2} \quad [4.2-7]$$

$$P_r = \frac{\mu_i \cdot c_i}{\lambda_i} \quad [4.2-8]$$

where,

ρ_i – density of the gas space

μ_i – dynamic viscosity of the gas space

c_i – specific heat capacity of the gas space

$T_{m,i}$ – mean temperature of the gas space, set to 283 K for all glazing cavities

ΔT_i – temperature difference between glass surfaces bounding the gas space

Gas fill properties (i.e. density, thermal conductivity, viscosity and specific heat capacity) are evaluated at the mean temperature of the gas space ($T_{m,i}$). The density is determined using the perfect gas law, while the other properties of individual gasses, used in sealed glazing units, are determined as a linear function of temperature. The properties of gas mixtures can be calculated as a function of corresponding properties of the individual constituents.

$$P = \sum_{i=1}^k P_i F_i \quad [4.2-9]$$

where,

P – relevant property (density, thermal conductivity, viscosity or specific heat capacity)

P_i – corresponding property of the individual gas in the gas mixture

F_i – volume fraction of the individual gas in the gas mixture

For double glazing systems the temperature difference ΔT_i has fixed value of 15 K. For the glazing systems with more than one gas space, which require iterative solution algorithm, the temperature difference ΔT_i is determined using following relations for the first and for all other iterations.

$$\Delta T_i = \frac{15}{N} \quad [4.2-10]$$

where,

N – number of glazing cavities (i.e. gas spaces)

$$\Delta T_i = 15 \frac{1/h_{s,i}}{\sum_{i=1}^N 1/h_{s,i}} \quad [4.2-11]$$

where,

$1/h_{s,i}$ – glazing cavity thermal resistance from the previous iteration

$\sum_{i=1}^N 1/h_{s,i}$ – overall thermal resistance of glazing cavities

In the first iteration, ΔT_i is calculated using equation [4.2-10], while in the second and all later iterations (if necessary), the equation [4.2-11] is used.

4.2.2. Radiation Conductance

The radiation conductance of the glazing cavity is defined as a function of mean temperature of the glazing cavity, as well as emissivities of the surfaces bounding the glazing cavity.

$$h_{rs,i} = 4\sigma \left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 \right)^{-1} T_{m,i}^3 \quad [4.2-12]$$

where,

$$\sigma = 5.6697 \cdot 10^{-8} \frac{W}{m^2 K^4} \text{ – Stefan – Boltzmann's constant}$$

$T_{m,i}$ – mean temperature of the glazing cavity (gas space)

ε_1 and ε_2 – emissivities of the surfaces bounding the glazing cavity

4.3. Calculation of U – factor

When the final value of overall thermal resistance of glazing cavities ($\sum_{i=1}^N 1/h_{s,i}$) is found, the total thermal conductance (h_t) of the glazing system is calculated as:

$$h_t = \frac{1}{\sum_{i=1}^N \frac{1}{h_{s,i}} + \sum_{j=1}^M d_j r_j} \quad [4.3-1]$$

where,

$$\sum_{j=1}^M d_j r_j \text{ – overall thermal resistance of glazing layers}$$

d_j – glazing layer thickness

r_j – glazing layer thermal resistance, which represents the reciprocal of thermal conductance – k_j

Then, the glazing system thermal transmittance (U – factor) is determined using the following relation:

$$U = \left(\frac{1}{h_e} + \frac{1}{h_t} + \frac{1}{h_i} \right)^{-1} \quad [4.3-2]$$

where,

h_e – outdoor heat transfer coefficient, defined as per section 4.1

h_i – indoor heat transfer coefficient, defined as per section 4.1

h_t – total glazing system thermal conductance